

CP SYMMETRY VIOLATION IN SCALAR SECTOR IN 2HDM AND 331 MODELS

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Dedictory

I dedicate this work to my wife, my greatest love, the reason and the bliss in my life, who gives color to my equations and smiles to my ideas, the beauty who give me the courage and happiness which I need every day in my job.

Para ti, Hermosa.

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Abstract

In order to understand some frameworks for CP Violation scenarios in the scalar sector, have been studied the Two Higgs Doublet Model (2HDM). Then, we consider the 331 models, which are extensions of the SM whose main property is the incorporation of a group symmetry $SU(3)$ in the electroweak sector. In particular was chosen a 331 model using its free parameter $\beta = 1/\sqrt{3}$.

In order to reproduce an scenarios for CP Violation in the scalar sector, we must to introduce a discrete symmetry into the scalar triplets, which exhibit a spontaneous CP Violation frame with just one indepent CP phase associated. Finally we rotated to mass states and was made an overview for some phenomenological aspects.

Keywords .

- *Discrete symmetries,*
- *CP symmetry Violation,*
- *2HDM,*
- *331 model,*
- *Complex Scalar triplets,*
- *Z_2 symmetry*

Introduction

Invariance under discrete symmetries is an important issue in the context of particle physics, due to the physical conservations in nature like the invariance of the Charge-Parity-Time transformation (CPT). Thus, it is necessary to have a detailed understanding for the CP violation phenomena into both the theoretical and experimental framework. Furthermore, one of the main motivation to study models beyond the standard model (SM) is for searching additional sources of CP violation in order to obtain rare decays related with baryogenesis processes.

Among the extensions of the SM, in this work, first, we study the Two Higgs Doublet Model (2HDM), which introduces a second identical complex scalar field. Second, we consider the 331 models, which are extensions of the SM whose main property is the incorporation of a group symmetry $SU(3)$ in the electroweak sector and the introduction of new particles that produce a wide range of new physics at the scale of energies currently being explored in high energy colliders.

In addition to obtain some insight into the CP features of the 2HDM and its realization, in this work we initiate a detailed theoretical study of the scalar sector of the 331 models in order to incorporate CP violation, such as the inclusion of complex scalar couplings in the Higgs potential or complex vacuum expectation values (VEV) in the scalar structure of the model to generate spontaneous CP violation. To address the above study, additional constraints such as continuous and discrete global symmetries must be proposed.

Although we can find in the literature many phenomenological studies of CP violation in extended models, new data reported by different collaborations demands a continuous updating of the previous studies, and motivates the proposal of new analyses that allow us to explain small deviations found in the data that can be associated to new physics, or to find further constraints of the parameters from theoretical models.

This work is organized as follows. First, in chapter 1, we present the basic concepts associated to discrete symmetries, specifically, the charge and parity symmetries. Second, in chapter 2,

an introduction of the 2HDM is shown, altogether with its CP properties and scenarios for CP violation. Chapter 3 is devoted to review the general properties of the CP conservative 331 model. Later, in chapter 4, we propose an extension of the 331 model with CP violation in the Higgs sector, including minimal conditions for its realization. In particular, the proposed model generate spontaneous CP violation. Rotations into mass eigenstates are obtained in chapter 5. Finally, we discuss some phenomenological prospects for future studies in order to test the predictions and consequences of the model.

CP Symmetry Transformations

In this chapter, a detailed understanding is obtained of how discrete symmetries transform complex scalar fields and how these symmetries are broken.

A discrete symmetry describes a non-continuous change in a system. Discrete symmetries in nature, sometimes involve some type of 'swapping' over some physical dimension. These swaps are usually called as reflections or interchanges. In particle physics there are three important discrete symmetries:

- Parity Symmetry
- Charge Conjugation
- Time Reversal

These symmetries have an associated transformation operator U , which must satisfy the Wigner Theorem[4, 5]. A transformation operator for any symmetry in a Hilbert space must be an unitary and linear operator or antiunitary and antilinear operator, it is:

Unitary and Linear:

$$\langle Ua|Ub \rangle = \langle a|b \rangle \quad , \quad U(|\zeta a + \eta b \rangle) = \zeta|Ua \rangle + \eta|Ub \rangle.$$

Antiunitary and Antilinear:

$$\langle Ua|Ub \rangle = \langle a|b \rangle^* \quad , \quad U(|\zeta a + \eta b \rangle) = \zeta^*|Ua \rangle + \eta^*|Ub \rangle.$$

Then, symmetries transformations must satisfy the following conditions:

I) The corresponding operator must be unitary:

$$U^\dagger U|a\rangle = |a\rangle \rightarrow UU^\dagger = U^\dagger U = 1. \quad (1.1)$$

II) Applying transformation twice leaves invariant the quantum state up to a unitary phase:

$$UU|a\rangle = \eta_a^2|a\rangle. \quad (1.2)$$

Eigenvalues must satisfy the unitary property (I) and the twice transformation property (II). Considering $\eta_a^2 = 1$, to remain the initial state $\eta_a^2|a\rangle = |a\rangle$, we get as consequence that operator satisfy hermiticity property $U^\dagger = U$, so operator has real eigenvalues [4, 5]:

$$UU|a\rangle = UU^\dagger|a\rangle, \quad (1.3)$$

$$\eta_a\eta_a|a\rangle = \eta_a\eta_a^*|a\rangle, \quad (1.4)$$

$$\eta_a = \eta_a^* = \pm 1. \quad (1.5)$$

In order to apply discrete symmetries transformations over the quantum fields, we need to express them as Fourier basis, using the notation presented in appendix A, and shown in table 1.1.

Table 1.1: Quantum fields - Fourier Basis

Dirac Field	$\Psi(x) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_p}} \sum_s (a_{p,s}u^s(p)e^{-ipx} + b_{p,s}^\dagger v^s(p)e^{ipx})$
Real Scalar Field	$\phi(x) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_p}} (a_p e^{-ipx} + a_p^\dagger e^{ipx})$
Complex Scalar Field	$\phi_c(x) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_p}} (a_p e^{-ipx} + b_p^\dagger e^{ipx})$
Vector Field	$A_\mu(p, x) = \int \frac{d^3p}{(2\pi)^3\sqrt{2\omega_p}} \sum_{\lambda=0}^3 (\epsilon_\mu(p, \lambda)a_{p,\lambda}e^{-ipx} + \epsilon_\mu^*(p, \lambda)a_{p,\lambda}^\dagger e^{ipx})$

1.1 Parity Symmetry

Parity transformation can be defined as the simultaneous flip in the sign of all three spatial coordinates. More formally¹, it is the transformation that reverse the momentum of a particle

¹Under parity transformation $(t, \vec{x}) \rightarrow (t, -\vec{x})$; the boost generators behave as true vectors and change sign ($\mathbf{K} \rightarrow -\mathbf{K}$), since the parity transformation reverses the velocity (\vec{v}) of the boost. The angular momentum generator is instead a pseudovector and then is unchanged ($\mathbf{J} \rightarrow \mathbf{J}$)[5, 1]

without flipping its spin [6]:

$$P|\Psi_{p,s}\rangle = \eta|\Psi_{-p,s}\rangle, \quad (1.6)$$

$$P|a\rangle = \eta_a|a_{-p,s}\rangle. \quad (1.7)$$

In order to have a mathematical definition for a generic multiparticle state, we introduce the parity transformation definition (P), applying over particles and antiparticles operators (\hat{a} and \hat{b}):

$$Pa_{\mathbf{p},s}P^\dagger = \eta_a a_{-\mathbf{p},s} \quad , \quad Pa_{\mathbf{p},s}^\dagger P^\dagger = \eta_a^* a_{-\mathbf{p},s}^\dagger, \quad (1.8)$$

$$Pb_{\mathbf{p},s}P^\dagger = \eta_b b_{-\mathbf{p},s} \quad , \quad Pb_{\mathbf{p},s}^\dagger P^\dagger = \eta_b^* b_{-\mathbf{p},s}^\dagger. \quad (1.9)$$

Applying parity transformation over the quantum fields (table 1.1), and using equations (1.8, 1.9), we can obtain how each field transform, as shown in the table 1.2. Different bilinear terms under Parity transformation, are shown in table 1.3.

Table 1.2: Quantum fields - Discrete Transformations [1]

Field \mathcal{F}	$P\mathcal{F}P$	Parity phase condition	Charge Conjugation $C\mathcal{F}C$
Dirac Field - Ψ h.c. - $\bar{\Psi}$	$\eta_a \gamma^0 \Psi(-x)$ $\eta_a^* \bar{\Psi}(-x) \gamma^0$	$\eta_a = -\eta_b^*$ $\eta_a^* = -\eta_b$	$-i\eta_c (\bar{\Psi} \gamma^0 \gamma^2)^T$ $-i\eta_c (\gamma^0 \gamma^2 \Psi)^T$
Complex Scalar Field - ϕ	$\eta_a \phi_c(-x)$	$\eta_a = \eta_b^*$	$\eta_c (\phi^\dagger)^T$
Vector Field - A_μ	$\eta_a A^\mu(-x)$	$\eta_a = \eta_a^*$	$-\eta_c A_\mu$

1.2 Charge Conjugation

Charge conjugation (C) transforms a particle into its antiparticle with the same mass, momentum and spin, but opposite quantum numbers like electric charge or baryon number[1]:

$$C|\psi\rangle = |\bar{\psi}\rangle, \quad C^\dagger|\bar{\psi}\rangle = |\psi\rangle. \quad (1.10)$$

Charge conjugation satisfy properties shown in equations 1.1 and 1.2, then we can assume real eigenvalues for its transformation operator. For a multiparticle state we can define the following transformation rules for charge conjugation:

$$Ca_{p,s}C = \eta_c b_{p,s} \quad , \quad Cb_{p,s}C = \eta_c a_{p,s}, \quad (1.11)$$

$$Ca_{p,s}^\dagger C = \eta_c b_{p,s}^\dagger \quad , \quad Cb_{p,s}^\dagger C = \eta_c a_{p,s}^\dagger. \quad (1.12)$$

Proceeding in the same way as parity transformation, we can show how all the quantum fields change under charge conjugation. The results are presented in the table 1.2, and the bilinear terms under Charge Conjugation are presented in table 1.3.

1.3 CP Symmetry

CP transformation is obtained when charge conjugation and parity transformation simultaneously are applying over certain system. CP symmetry states that the physics laws should be the same if a particle is interchanged with its antiparticle, and simultaneously its spatial coordinates are inverted. There is another discrete symmetry; *time reversal*, However, due to the CPT theorem, T transformation is equivalent to CP transformation.

CPT theorem

CPT transformation is the simultaneous change of charge conjugation (C), parity transformation (P), and time reversal (T). The CPT theorem says that any Lorentz invariant local quantum field theory with a hermitian Hamiltonian must have an exact CPT symmetry [7, 4, 1].

This important theorem can be proven rigorously in axiomatic field theory based on the assumptions of[1]:

- Lorentz invariance
- the existence of a unique vacuum state
- weak local commutativity obeying the ‘right’ statistics

Bilinear Terms

There are some bilinear terms that can exhibit or not an invariance under discrete transformations. For a general bilinear term of the form $\bar{\psi}\Gamma\psi$, we show in Table 1.3 how each discrete symmetry transform the coupling.

Table 1.3: Fermion bilinear terms transformations[1]

Γ	Γ^P $\gamma^0\Gamma\gamma^0$	Γ^C $(\gamma^2\gamma^0\Gamma\gamma^2\gamma^0)^T$	Γ^T $-\gamma^1\gamma^3\Gamma^*\gamma^1\gamma^3$	Γ^{CP} $-(\gamma^2\Gamma\gamma^2)^T$	Γ^{CPT} $(\gamma^5\gamma^0\Gamma^\dagger\gamma^0\gamma^5)^T$
1	1	1	1	1	1
γ^5	$-\gamma^5$	γ^5	γ^5	$-\gamma^5$	$-\gamma^5$
γ_μ	γ^μ	$-\gamma_\mu$	γ^μ	$-\gamma^\mu$	$-\gamma_\mu$
$\gamma_\mu\gamma^5$	$-\gamma^\mu\gamma^5$	$\gamma_\mu\gamma^5$	$\gamma^\mu\gamma^5$	$-\gamma^\mu\gamma^5$	$-\gamma_\mu\gamma^5$
$\sigma_{\mu\nu}$	$\sigma^{\mu\nu}$	$-\sigma_{\mu\nu}$	$-\sigma^{\mu\nu}$	$-\sigma^{\mu\nu}$	$\sigma_{\mu\nu}$
$\sigma_{\mu\nu}\gamma^5$	$-\sigma^{\mu\nu}\gamma^5$	$-\sigma_{\mu\nu}\gamma^5$	$-\sigma^{\mu\nu}\gamma^5$	$\sigma^{\mu\nu}\gamma^5$	$-\sigma_{\mu\nu}\gamma^5$
∂_μ	∂^μ	∂_μ	$-\partial^\mu$	∂^μ	$-\partial_\mu$

CP violation

CP symmetry is an exact symmetry for multiparticle systems, except for neutral K & B mesons, which are experimentally supported. The discovery of CP violation was observed in 1964 in the decays of neutral kaons by James Cronin and Val Fitch. In the context of SM; Kobayashi and Maskawa (KM) proposed three generations of quarks to produce one irreducible phase accounting for the CP violation.

According to the three² conditions proposed by Sakharov³, a theory of particles must have a CP violation phase in order to explain annihilation processes between matter and antimatter (baryogenesis), which is observed in nature. Since the KM phase can not explain CP violation at baryogenesis scales, new sources of CP violation must appear in extended models.

1.4 Symmetry Breaking Types

Symmetry breaking can occur in two different ways. In particular, if a system exhibits a discrete symmetry, this must obey the following fundamental properties:

- (1) The groundstate or ‘vacuum’ remains invariant:

$$P|0\rangle = |0\rangle, \quad C|0\rangle = |0\rangle, \quad T|0\rangle = |0\rangle. \quad (1.13)$$

- (2) The action, Lagrangian or Hamiltonian also remains invariant:

$$[P, H] = [C, H] = [T, H] = 0. \quad (1.14)$$

$$\text{Or: } S = \int d^4x \mathcal{L}(t, \vec{x}) \rightarrow S' = S.$$

- (3) The quantization conditions must remain invariant.

When the symmetries are broken, one of the above conditions are not satisfied. There are two ways to do this:

Explicit Breaking

If condition (2) is not valid, CP invariance is broken explicitly by introducing CP-Violating terms. In some cases, CP violating parameters may generate relevant dynamical quantities. CP phase in the CKM matrix is an example for explicit CP violation. Other example of explicit breaking in the Higgs potential for our interest is presented in the appendix B.

² 1) Baryon number violation, 2) C and CP symmetry violation, 3) interactions out of thermal equilibrium

³ Andrei Sakharov proposed baryogenesis conditions in 1967

Spontaneous Breaking

If condition (1) is not valid, CP symmetry is realized in a spontaneous fashion. In this case, the Lagrangian conserves CP (the gauge and Yukawa couplings can be made real), but the ground state does not, then vacuum expectation value of the neutral Higgs fields develop complex phases. Again, usually for extended models, the relevant quantities can be derived from the dynamics.

CP Violation in the 2HDM context

The Two Higgs Doublet Model (2HDM) is based on two $SU(2)$ complex doublets of scalar fields with the same hypercharge. The best motivation of these extended models is the minimal supersymmetric extension of the Standard Model (MSSM) [8], which requires a second Higgs doublet in order to preserve the cancellation of gauge anomalies. Originally, this model was motivated for CP violation studies.

2.1 Model Review

In the 2HDM there are two vacuum expectation values (VEV's), one for each doublet, which could have a relative complex phase between them. That's the reason that allow us generate a source for a spontaneous CP violation coming from the electroweak spontaneous symmetry breaking. Furthermore, if there are complex parameters in the scalar or Yukawa sector, explicit CP violation also may arise [9].

2HDM structure

The 2HDM is one of the simplest extensions of the Higgs sector in the Standard Model (SM), where two scalar doublets with the same quantum numbers (Hypercharge $Y_1 = Y_2 = 1$) are considered:

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix} ; \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}, \quad (2.1)$$

As in the SM, both doublets can acquire a vacuum expectation value, however in the 2HDM, the vacuum structure is much richer than in the SM. There are three scenarios for the vacuum state. In general, the vacuum states have the form:

$$\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu_1 \end{pmatrix} ; \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \mu \\ \nu_2 e^{i\delta} \end{pmatrix}. \quad (2.2)$$

The three scenarios for the VEV are:

Neutral real vacuum: is the case when $\mu = 0$ and $\delta = 0$ and there is not spontaneous CP symmetry breaking.

Neutral complex vacuum: taking $\mu = 0$ and considering non nule phase we can obtain spontaneous CP breaking.

Charged vacuum: considering $\mu \neq 0$ and $\delta = 0$, we have electromagnetic charge breaking without spontaneous CP violation.

Choosing the neutral complex vacuum state, which is of our interest, with a convenient parametrization, we have:

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{h_1 + \nu_1 + ig_1}{\sqrt{2}} \end{pmatrix} ; \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{h_2 + \nu_2 e^{i\delta} + ig_2}{\sqrt{2}} \end{pmatrix}. \quad (2.3)$$

Into the most general Lagrangian for the 2HDM, there are additional terms in relation to the minimal SM in the scalar potential, the kinetic sector and the Yukawa interactions.

Yukawa Sector

Another motivation for the 2HDM is the hierarchy mass problem. At the fermionic sector, in the third generation of quarks; the experimental data show a large top-bottom ratio $m_t/m_b \approx 35$, which is not understood in the context of the SM. In 2HDM, we have the freedom to choose which scalar field couple with Down or Up quarks, for example the quark bottom can receive its mass from one doublet (such as Φ_1) and the quark top from another doublet (such as Φ_2), each one with different VEV, which may explain the mass difference.

Table 2.1: 2HDM types for yukawa sector[2, 3]

Model	U_{iR}	D_{iR}	E_{iR}
Type I	Φ_2	Φ_2	Φ_2
Type II	Φ_2	Φ_1	Φ_1
Lepton Specific	Φ_2	Φ_2	Φ_1
Flipped	Φ_2	Φ_1	Φ_2

The most general Yukawa Lagrangian that couples the Higgs fields with fermions is written

as:

$$-\mathcal{L}_Y = \bar{Q}_{iL}(\eta_{ij}^u \tilde{\Phi}_1 + \xi_{ij}^u \tilde{\Phi}_2)U_{jR} + \bar{Q}_{iL}(\eta_{ij}^d \Phi_1 + \xi_{ij}^d \Phi_2)D_{jR} + \bar{l}_{iL}(\eta_{ij}^e \Phi_1 + \xi_{ij}^e \Phi_2)E_{jR} + h.c., \quad (2.4)$$

where indices (i, j) denotes flavor families, $\Phi_{1,2}$ are the complex scalar doublets, where $\tilde{\Phi}_{1,2} = i\sigma_2 \Phi_{1,2}$, and the Yukawa couplings parameters are η and ξ , which can be represented as 3×3 matrices, (fields are not in mass eigenstates yet). Depending of which scalar doublet each quark type couple, 2HDM can be classified in different models, as shown in table 2.1.

Higgs Kinetic Terms

The kinetic Higgs Lagrangian of the SM can be extended to Two Doublets in the following form:

$$\mathcal{L}_K = (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2). \quad (2.5)$$

This term maintain the same covariant derivative as SM for the left handed chirality:

$$D_\mu = \partial_\mu + \frac{ig}{2} \vec{W}_\mu \cdot \sigma + \frac{ig'}{2} Y B_\mu.$$

Due to gauge invariance and self-hermeticity, kinetic terms does not change under Higgs basis transformation, as explained in section 2.2. Also the potential is invariant under charge conjugation and other discrete or global symmetries.

After diagonalization to obtain the mass eigenstates, the physical gauge bosons are:

$$W_\mu^\pm = \frac{W_\mu^1 \mp iW_\mu^2}{\sqrt{2}}, \quad (2.6)$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_W & -\sin \theta_W \\ \sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}, \quad (2.7)$$

where θ_W is the Weinberg mixing angle, which is given by $\tan \theta_W = \frac{g'}{g}$. The mass expressions are:

$$M_{W^\pm}^2 = \frac{1}{4}g^2(\nu_1^2 + \nu_2^2), \quad (2.8)$$

$$M_Z^2 = \frac{1}{4}(\nu_1^2 + \nu_2^2)(g'^2 + g^2). \quad (2.9)$$

As we can see, in mass expressions, both VEV contribute in the same way. Furthermore, we can define the electroweak VEV; $\nu_{ew}^2 = \nu_1^2 + \nu_2^2$, such that the mass expressions have the same form as in the SM.

Higgs Potential

The scalar sector of the 2HDM has many interesting features. For example, in addition to CP breaking,¹ electromagnetic charge breaking can also be generated, providing mass to photons. Also, it is possible to have inert vacuum, in which one of the neutral scalars does not couple to gauge bosons, then CP symmetry and electromagnetic charge are preserved.

In general, the Higgs potential has 14 independent parameters. However, the two Higgs doublets are not physical observables; only the scalar mass eigenstates are physical particles. In addition, since both doublets are identical (same quantum numbers), we have the freedom to redefine the basis for the scalar fields. This change of basis allow us to absorb some parameters from the potential. Likewise in the Yukawa sector, if we only impose gauge invariance, the renormalizable potential is not unique. In the literature we find three different notation for Higgs potential [3], however for our purpose we are going to use the following notation:

$$\begin{aligned}
 V_H = & - \mu_1^2 \hat{A} - \mu_2^2 \hat{B} - \mu_3^2 \hat{C} - \mu_4^2 \hat{D} + \lambda_1 \hat{A}^2 + \lambda_2 \hat{B}^2 + \lambda_3 \hat{C}^2 + \lambda_4 \hat{D}^2 \\
 & + \lambda_5 \hat{A}\hat{B} + \lambda_6 \hat{A}\hat{C} + \lambda_8 \hat{A}\hat{D} + \lambda_7 \hat{B}\hat{C} + \lambda_9 \hat{B}\hat{D} + \lambda_{10} \hat{C}\hat{D},
 \end{aligned}
 \tag{2.10}$$

where the hermitian gauge invariant operators are:

$$\hat{A} = \Phi_1^\dagger \Phi_1, \quad \hat{B} = \Phi_2^\dagger \Phi_2.
 \tag{2.11}$$

$$\hat{C} = \text{Re}(\Phi_1^\dagger \Phi_2) = \frac{1}{2}(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1).
 \tag{2.12}$$

$$\hat{D} = \text{Im}(\Phi_1^\dagger \Phi_2) = \frac{-i}{2}(\Phi_1^\dagger \Phi_2 - \Phi_2^\dagger \Phi_1).
 \tag{2.13}$$

This notation is convenient for studies of features such as the existence and number of minima of the scalar potential, also it is convenient for CP studies. Since the Yukawa couplings do not involve bilinear terms with the Higgs doublets, this notation can not be applied for the Yukawa sector.

2.2 CP Transformation

Basis Transformation

The scalar doublets proposed in the potential (2.10) are not physical; only the scalar mass eigenstates corresponds to physical particles. Furthermore, any combination of the doublets which respects the symmetries of the theory will produce the same physical predictions. Any combination of Φ_1, Φ_2 can represents a basis for the doublets. We may rewrite the potential in terms of new doublets Φ'_a , obtained from the original ones by a global basis transformation, which is written as:

¹2HDM was motivated to find CP violation additional sources in 1973 with T. Lee proposal [10]

$$\Phi'_a = \sum_{b=1}^2 U_{ab} \Phi_b. \quad (2.14)$$

Consequently, the parameters in the potential also transforms through the unitary matrix (U). This basis transformation will be useful in order to reduce the number of parameters of the Higgs potential. Using a basis transformation it is possible to reduce from 14 free parameters to 11 physical parameters.

In particular, CP invariance is manifested depending on the scalar basis. Specifically, CP violating effects are absent if and only if there exists a basis in which the two vacuum expectation values and all scalar potential parameters are simultaneously real [2, 3].

It is important to point out that this feature in the two Higgs doublet model does not appear in the 331 model, because there are two triplets involved in the transition in which the electroweak symmetry is broken to electromagnetism but, unlike the 2HDM, those two triplets are not identical; they differ from each other by a new X quantum number. Thus, in 331 model it is not necessary to set a basis transformation.

GCP transformations

The usual CP transformation of a complex scalar field, is shown in the table 1.2. However, in the presence of identical doublets, the possibility of arbitrary basis transformations should be included in the definition of the CP transformation. Then, we must consider a more general version of the CP transformation, which we denote as ‘GCP’:

$$\begin{aligned} \Phi_a &\rightarrow (\Phi'_a)^{GCP} = \sum_{b=1}^2 X_{ab} \Phi_b^*, \\ \Phi_a^\dagger &\rightarrow (\Phi'^\dagger_a)^{GCP} = \sum_{b=1}^2 X_{ab}^* \Phi_b^T, \end{aligned}$$

where X is also an unitary matrix, which can be applied over the potential parameters, in the same way as the basis transformation. GCP transformations were first discussed by Lee and Wick [11]. Discussions in the scalar sector were developed by the Vienna group [8].

2.3 Explicitly CP violation

CP violation may be either explicit or spontaneous. As we mentioned before; explicit CP violation occurs when terms into the Lagrangian are non invariant under CP transformation. In particular, those terms can generate an irreducible complex phase that remains in the

Lagrangian, even after applying a rotations and rephasing over the scalar doublets.

The most general potential in 2HDM (eq. (2.10)) explicitly violates the CP symmetry. As we mentioned before, CP invariance requires a scalar basis in which all potential parameters are real.

2.3.1 Global symmetries

In general, given an arbitrary potential, the existence or not of a real basis may be difficult to discern. In order to find the necessary conditions for CP conserving at the Lagrangian level, we start from the potential (eq. (2.10)), which explicitly violates CP.

Under Charge Conjugation the scalar fields transform as: $\Phi_i^\dagger \Phi_j \rightarrow e^{i(\alpha_j - \alpha_i)} \Phi_j^\dagger \Phi_i$. For instance if we choose $\alpha_i = \alpha_j$, the gauge operator \hat{D} (eq. (2.13)) changes with a minus sign. Thus, if we assume that the Higgs potential holds a charge conjugation invariance, the number of parameters reduces to ten:

$$V_H = -\mu_1^2 \hat{A} - \mu_2^2 \hat{B} - \mu_3^2 \hat{C} + \lambda_1 \hat{A}^2 + \lambda_2 \hat{B}^2 + \lambda_3 \hat{C}^2 + \lambda_4 \hat{D}^2 + \lambda_5 \hat{A} \hat{B} + \lambda_6 \hat{A} \hat{C} + \lambda_7 \hat{B} \hat{C}, \quad (2.15)$$

where the non-invariant parameters were removed: $\mu_4 = \lambda_{8,9,10} = 0$. The charge symmetry invariance is equivalent to CP invariance for this potential, since all fields are scalars. However, this potential could still induce spontaneous CP violation [12]. However, there are two natural ways to guarantee complete CP invariance. The first one consists to impose invariance under a Z_2 symmetry, in which it is possible to transform to a real basis; and the other one is a Global symmetry.

Z_2 Symmetry

We define this transformation as: $\Phi_1 \rightarrow \Phi_1$, $\Phi_2 \rightarrow -\Phi_2$. Under this transformation, the Higgs potential forbids 3 more terms $\mu_3 = \lambda_{6,7} = 0$, obtaining:

$$V_{Z_2} = -\mu_1^2 \hat{A} - \mu_2^2 \hat{B} + \lambda_1 \hat{A}^2 + \lambda_2 \hat{B}^2 + \lambda_3 \hat{C}^2 + \lambda_4 \hat{D}^2 + \lambda_5 \hat{A} \hat{B}. \quad (2.16)$$

In order to generate spontaneous CP Violation, we can introduce a soft break term by admitting a quadratic term that violates the symmetry. In that way we can consider $\mu_3 \neq 0$; thus we have the following potential:

$$V'_{Z_2} = V_{Z_2} - \mu_3^2 \hat{C}. \quad (2.17)$$

Global $U(1)$ Symmetry

We define this transformation as $\Phi_2 \rightarrow e^{i\xi} \Phi_2$. Under this transformation, again, the Higgs potential forbids 3 terms $\mu_3 = \lambda_{6,7} = 0$ but in addition two terms must be equal $\lambda_3 = \lambda_4$, so the potential is:

$$V_{U(1)} = -\mu_1^2 \hat{A} - \mu_2^2 \hat{B} + \lambda_1 \hat{A}^2 + \lambda_2 \hat{B}^2 + \lambda_3 (\hat{C}^2 + \hat{D}^2) + \lambda_5 \hat{A} \hat{B}. \quad (2.18)$$

In the same way as with Z_2 symmetry, we can introduce the soft break term to obtain a CP-violating Higgs potential.

2.4 Spontaneously CP violation

Spontaneous CP breaking in 2HDM was suggested by T. D. Lee [10] at the initial stages of unified gauge theories. In order to have spontaneous CP violation, we must consider a Lagrangian which is explicitly CP conserving, but after spontaneous gauge symmetry breaking the vacuum generate CP violation [8, 2]. As we showed previously, there are different types of vacuum, so we must to be careful to choose the correct one in order to obtain CP breaking. Also we have to check if the Lagrangian is explicitly CP invariant under any GCP (remember that CP transformation depends on the scalar doublets basis). In order to have a genuine spontaneous CP violation, the following two conditions must be satisfied [2]:

- The Lagrangian is invariant under a CP transformation.
- There is no transformation, which can be physically interpreted as CP transformation, which leaves both the vacuum and the Lagrangian invariant.

Several articles proposed some invariants obtained from the parameters of the potential to discern between an spontaneous or explicit violation [13, 14]. In practice, distinguishing between explicit and spontaneous CP violation by experimental observations and analysis seems extremely difficult. We must ensure that a spontaneous CP violation should be able to reproduce the phase of CKM matrix, and also must avoid large contributions to FCNC processes [2].

As we explained in the previous section, the Higgs potentials (2.17) including the soft breaking term $\mu_3 \neq 0$ is a potential that violates the CP symmetry in a spontaneous way.

2.5 Mass Eigenstates

Given a stationary point for the Higgs potential, it is necessary to determine if this is a minimum or not, so one needs to analyze the second derivative of the potential. As a result, we obtain the scalar mass matrices:

$$M_{ij}^2 = \left. \frac{\partial^2 V_H}{\partial \Phi_i \partial \Phi_j} \right|_{\Phi_i=0}.$$

The Higgs masses and Higgs eigenstates are defined in terms of the parameters μ_i , λ_i from the potential, and consequently depends of this. In order to get the Higgs masses eigenstates, we must to diagonalize the mass matrices.

CP conservative case

The CP conservative case corresponds to the Higgs potential with one of the global symmetries. Taking for example, the Z_2 symmetry potential (eq. (2.16)), we show the scalar spectrum. Considering in general the VEV in equation (2.2 with $\mu = 0$) (i.e. the CP neutral vacuum), the Higgs sector consists of the following spectrum:

- 2 CP even scalars (H^0, h^0),
- 1 CP odd scalar (A^0),
- 2 Charged Higgs bosons (H^\pm),
- 3 Goldstone bosons (G^\pm, G^0).

So, in the 2HDM appears 4 new Higgs bosons $\{H^+, H^-, H^0, A^0\}$: two charged and two neutral bosons. The mass eigenstates, can be obtained from the weak eigenstates through the following rotation matrices [3]:

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}, \quad (2.19)$$

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}, \quad (2.20)$$

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} g_1 \\ g_2 \end{pmatrix}, \quad (2.21)$$

where the CP even fields and CP odd appears separated from each other, and diagonalize independently with two rotation angles, defined as:

$$\tan \beta = \frac{\nu_2}{\nu_1}, \quad \tan 2\alpha = \frac{2\tilde{\lambda}\nu_1\nu_2}{\lambda_1\nu_1^2 + \lambda_2\nu_2^2}.$$

The Higgs masses for this particular model, reads [3]:

$$m_{H^\pm}^2 = -\lambda_3(\nu_1^2 + \nu_2^2), \quad (2.22)$$

$$m_{A^0}^2 = \frac{1}{2}(\lambda_4 - \lambda_3)(\nu_1^2 + \nu_2^2), \quad (2.23)$$

$$m_{H^0, h^0}^2 = \lambda_1\nu_1^2 + \lambda_2\nu_2^2 \pm \sqrt{(\lambda_1\nu_1^2 - \lambda_2\nu_2^2)^2 + 4\nu_1^2\nu_2^2\tilde{\lambda}}, \quad (2.24)$$

with:

$$\tilde{\lambda} = \frac{1}{2}(\lambda_3 + \lambda_5). \quad (2.25)$$

CP non conservative case

Considering the weak basis representation shown in equation (2.3) and introducing the soft breaking term into the potential (eq. (2.17)), we can obtain a different phenomenology in the scalar sector. In this case, as a result of the additional CP violating terms, we obtain the mixing between CP even and CP odd fields, as follows.

For the charged sector we have $H^\pm = -\sin\beta\phi_1^\pm + \cos\beta\phi_2^\pm$. If we define the field $A^0 = -\sin\beta g_1 + \cos\beta g_2$, the rotation into mass eigenstates is:

$$\begin{pmatrix} H_1^0 \\ H_2^0 \\ H_3^0 \end{pmatrix} = \mathbb{R} \begin{pmatrix} h_1 \\ h_2 \\ A^0 \end{pmatrix}, \quad (2.26)$$

where the matrix rotation is composed by the Euler angles (α_i). Using $c_i \equiv \cos\alpha_i$, $s_i \equiv \sin\alpha_i$, we have:

$$\mathbb{R} = \mathbb{R}_3\mathbb{R}_2\mathbb{R}_1 = \begin{pmatrix} c_1c_2 & c_2s_1 & s_2 \\ -c_1s_2s_3 - c_3s_1 & c_1c_3 - s_1s_2s_3 & c_2s_3 \\ -c_1c_3s_2 + s_3s_3 & -c_1s_3 - c_3s_1s_2 & c_2c_3 \end{pmatrix}. \quad (2.27)$$

After the diagonalization process, we find mass terms that mix all the fields in this basis, obtaining couplings that violate CP symmetry [15]. The second and third angles contains the complex CP violating phase as:

$$s_2 \approx \delta \frac{\cos(\beta + \alpha)}{M_A^2 - M_h^2} \nu^2, \quad s_3 \approx -\delta \frac{\sin(\beta + \alpha)}{M_A^2 - M_H^2} \nu^2. \quad (2.28)$$

CP Conservative - 331 model

Essentially the 331 model is a model with $SU(3)_L \times U(1)_X$ gauge symmetries in the electroweak sector. This group is spontaneously broken to the SM with $SU(2)_L \times U(1)_Y$ gauge groups. Subsequently we must break the SM local gauge groups to the electromagnetic ($U(1)_Q$) group. Therefore, the 331 model has an extended Higgs sector [16], where the first transition occurs at a higher scale than the electroweak breaking.

In the next sections we are going to discuss a specific 331 model with $\beta = \frac{1}{\sqrt{3}}$ ¹.

The 331 model introduces new particles in the fermionic sector, which may introduce new physics. The properties for this particles are presented in section 3.2.1. In addition, the extension of the SM gauge group from $SU(2)_L$ to $SU(3)_L$ implies the existence of 5 new gauge bosons.

3.1 Gauge Symmetries and Structure

Including the strong sector, we can construct a quiral model that exhibits gauge local symmetry under the groups: $SU(3)_C \times SU(3)_L \times U(1)_X$, also known as *331 model*. However the color sector is the same as the Minimal Standard Model, then we make emphasis in the gauge group $SU(3)_L \times U(1)_X$.

3.1.1 Group Generators

The $SU(3)_L$ group must satisfy the Lie algebra, thus the generators are proportional to the Gell Mann matrices λ_i :

¹The parameter β will be introduced in section 3.1.1 (eq. 3.4)

$$G_\alpha = \frac{1}{2}\lambda_\alpha.$$

Also the generators will satisfy the commutator relations for special unitary groups:

$$[G_\alpha, G_\beta] = if^{\alpha\beta\gamma}G_\gamma, \quad (3.1)$$

where f are structure constants which are skew-symmetric². Additional to the commutator relations, the generators also must satisfy the anticommutator relation:

$$\{G_\alpha, G_\beta\} = \frac{1}{3}\delta^{\alpha\beta} + d^{\alpha\beta\gamma}G_\gamma, \quad Tr(G_\alpha G_\beta) = \frac{1}{2}\delta_{\alpha\beta}. \quad (3.2)$$

The U(1) group, has one diagonal generator $G_0 = \frac{1}{\sqrt{6}}\mathbb{I}$. Then, after spontaneous symmetry breaking, we define the charge operator as function of diagonal generators $\hat{Q} = \alpha G_3 + \beta G_8 + \gamma G_0$. Redefining in terms of a new charge $x = \frac{\gamma}{\sqrt{6}}$, $X = x\mathbb{I}$, we have:

$$Q = \alpha G_3 + \beta G_8 + X. \quad (3.3)$$

Charge Operator

Above, we introduced the new quantum number (X). Also the \hat{Q} operator must satisfy the Gell-Mann Nijishima relation: $Q = G_3 + Y$, where G_3 is an extension of the third Pauli matrix σ_3 . Therefore, we choose $\alpha = 1$ in equation 3.3 and $Y = \beta G_8 + X$, leaving the parameter β as a free parameter of the 331 model. The electric charges are:

$$Q = \begin{pmatrix} \frac{1}{2} + \frac{\beta}{2\sqrt{3}} + X & 0 & 0 \\ 0 & -\frac{1}{2} + \frac{\beta}{2\sqrt{3}} + X & 0 \\ 0 & 0 & -\frac{\beta}{\sqrt{3}} + X \end{pmatrix}. \quad (3.4)$$

Hypercharge Operator

In a similar way, we can deduce the hypercharge operator, using the commutation relation, in order to obtain an operator as function of β parameter and X quantum number:

$$Y = \begin{pmatrix} \frac{\beta}{2\sqrt{3}} + X & 0 & 0 \\ 0 & \frac{\beta}{2\sqrt{3}} + X & 0 \\ 0 & 0 & -\frac{\beta}{\sqrt{3}} + X \end{pmatrix}. \quad (3.5)$$

²The elements for the structure constants which are not zero, are presented in the appendix C.

We can obtain a different particle spectrum for fermions and bosons, therefore a different phenomenology over the 331 model, depending on the value assigned to the β parameter. In particular, in order to not generate exotic charges,³ the β parameter must take the value:

$$\beta = \frac{1}{\sqrt{3}}. \quad (3.6)$$

Quiral Anomalies

The renormalization procedure, which is fundamental for any gauge theory, must be consistent with the symmetries of the model; those are related with the current conservation by the Noether's theorem [17]. This renormalization for the gauge symmetries at different corrections, is realized through the Ward-Takahashi identities. Specifically, the vertex corrections among gauge bosons are proportional to the coefficient of anomaly, which is defined as [17]:

$$A_{\alpha\beta\gamma} = 2 \sum_{\text{repes}} \text{Tr} \left(\{G_{\alpha}(\hat{T}_{q,\ell})_L, G_{\beta}(\hat{T}_{q,\ell})_L\} G_{\gamma}(\hat{T}_{q,\ell})_L - \{G_{\alpha}(\hat{T}_{q,\ell})_R, G_{\beta}(\hat{T}_{q,\ell})_R\} G_{\gamma}(\hat{T}_{q,\ell})_R \right),$$

where G_{α} are the group generators, T are the fermionic representations, while “ L ” denotes left handed chirality and “ R ” denotes right handed chirality. This coefficient must be zero to ensure the renormalization. This coefficient must be computed for both leptonic and the quark sector for each gauge symmetry, including the gravitational interaction.

In 331 models, the anomaly cancellation occurs only if there are three generations. In particular, the condition of cancellation of anomalies in 331 models impose the constrain that the number of generations must be equal to the number of colors, thus providing an explanation for the existence of 3 generations.

3.2 Particle Spectrum

As mentioned previously; the spectrum of particles in 331 model is extended. In order to be consistent with the SM phenomenology, these new particles must acquire mass at a very high energy. This can be achieved in a natural way by introducing a spontaneous symmetry breaking scheme in two stages at different energy scales, as explained in section 3.2.3.

3.2.1 Fermionic Spectrum

In 331 models, the left handed leptons come in triplets and the right handed in singlet representations, similar to the SM, while in the quark sector, two left handed generation comes in triplets while the other family has an anti-triplet representation in order to cancel anomalies.

³Exotic means; an electric charge that have not been seen in nature

Thus, the model is naturally non universal of families, which provide a hint to understand the hierarchy mass problem.

Table 3.1 shows the fermionic content for each family with their charges: $U(1)_X$ charge, electric charge (Q_{em}) and hypercharge (Y), where $J_{1,2,3}$ and $L_{1,2,3}$ are the new quark and lepton flavors.

Table 3.1: Fermionic Spectrum for 331 model with $\beta = \frac{1}{\sqrt{3}}$, ($SU(3)_C$, $SU(3)_L$, $U(1)_X$)

Fermionic Fields - Families			$\beta = \frac{1}{\sqrt{3}}$			
	I	II	III	(C, L, X)	Q_{em}	Y
Quarks	$\begin{pmatrix} u \\ d \\ J_1 \end{pmatrix}_L$	$\begin{pmatrix} c \\ s \\ J_2 \end{pmatrix}_L$		$(3, 3, 0)$	$\begin{pmatrix} 2/3 \\ -1/3 \\ -1/3 \end{pmatrix}$	$\begin{pmatrix} 1/6 \\ 1/6 \\ -1/3 \end{pmatrix}$
			$\begin{pmatrix} b \\ -t \\ J_3 \end{pmatrix}_L$	$(3, 3^*, \frac{1}{3})$	$\begin{pmatrix} -1/3 \\ 2/3 \\ 2/3 \end{pmatrix}$	$\begin{pmatrix} 1/6 \\ 1/6 \\ 2/3 \end{pmatrix}$
	u_R	c_R	t_R	$(3, 1, \frac{2}{3})$	$2/3$	$2/3$
	d_R	s_R	b_R	$(3, 1, -\frac{1}{3})$	$-1/3$	$-1/3$
	J_R^1	J_R^2		$(3, 1, -\frac{1}{3})$	$-1/3$	$-1/3$
			J_R^3	$(3, 1, \frac{2}{3})$	$2/3$	$2/3$
Leptons	$\begin{pmatrix} e \\ -\nu_e \\ L_1 \end{pmatrix}_L$	$\begin{pmatrix} \mu \\ -\nu_\mu \\ L_2 \end{pmatrix}_L$	$\begin{pmatrix} \tau \\ -\nu_\tau \\ L_3 \end{pmatrix}_L$	$(1, 3^*, -\frac{1}{3})$	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -1/2 \\ 1/2 \\ 0 \end{pmatrix}$
	e_R	μ_R	τ_R	$(1, 1, -1)$	-1	-1
	L_R^1	L_R^2	L_R^3	$(1, 1, 0)$	0	0

3.2.2 Bosonic Spectrum

In order to construct the bosonic spectrum, it is necessary to introduce the covariant derivative for the model. The definition for the electroweak sector is written as:

$$(D_\mu)_L = \partial_\mu - igW_\mu^\alpha G_\alpha - ig_X X B_\mu, \quad (D_\mu)_R = \partial_\mu - ig_X X B_\mu. \quad (3.7)$$

We must introduce 8 vector fields: W_μ^a , associated with $SU(3)$ generators and another one: B_μ , for the $U(1)$ group. Then using the commutator relation presented in equation (3.1) ($[G_\alpha, W_\mu^\beta G_\beta] = -(if_{\alpha\beta\gamma})W_\mu^\beta G_\gamma$), we have:

$$W_\mu = \frac{1}{2} \begin{pmatrix} W^3 + \frac{1}{\sqrt{3}}W^8 & W^1 - iW^2 & W^4 - iW^5 \\ W^1 + iW^2 & -W^3 + \frac{1}{\sqrt{3}}W^8 & W^6 - iW^7 \\ W^4 + iW^5 & W^6 + iW^7 & -\frac{2}{\sqrt{3}}W^8 \end{pmatrix}. \quad (3.8)$$

After applying the charge operator \hat{Q}

$$\hat{Q}W_\mu = G_3W_\mu + \beta G_8W_\mu + XW_\mu,$$

we obtain for $\beta = \frac{1}{\sqrt{3}}$:

$$Q_W = \frac{1}{2} \begin{pmatrix} 0 & 1 & \frac{1}{2} + \frac{\sqrt{3}\beta}{2} \\ -1 & 0 & -\frac{1}{2} + \frac{\sqrt{3}\beta}{2} \\ -\frac{1}{2} - \frac{\sqrt{3}\beta}{2} & \frac{1}{2} - \frac{\sqrt{3}\beta}{2} & 0 \end{pmatrix} \rightarrow Q_W = \frac{1}{2} \begin{pmatrix} 0 & 1 & 1 \\ -1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}. \quad (3.9)$$

We can define the gauge bosons representation for $SU(3)$, as:

$$U_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2) \quad , \quad V_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^4 \mp iW_\mu^5),$$

$$V_\mu^0 = \frac{1}{\sqrt{2}}(W_\mu^6 - iW_\mu^7) \quad , \quad \bar{V}_\mu^0 = \frac{1}{\sqrt{2}}(W_\mu^6 + iW_\mu^7),$$

where we can see that there are new charged and neutral vector bosons, so:

$$W_\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}(W^3 + \frac{1}{\sqrt{3}}W^8) & U_\mu^+ & V_\mu^+ \\ U_\mu^- & \frac{1}{\sqrt{2}}(-W^3 + \frac{1}{\sqrt{3}}W^8) & V_\mu^0 \\ V_\mu^- & \bar{V}_\mu^0 & -\frac{\sqrt{2}}{\sqrt{3}}W^8 \end{pmatrix}. \quad (3.10)$$

Additionally, for the $U(1)$ group, we have: $\mathbb{B}_\mu = \mathbb{I}B_\mu$ and $\hat{Q}_{B_\mu} = 0$, obtaining three gauge fields with charges equal to zero: W_μ^3, W_μ^8, B_μ , which in their mass eigenstates basis, correspond to the photon, and two neutral weak bosons Z and Z' .

3.2.3 Scalar sector

This sector contains interesting phenomenology through the scalar fields which present the scenario for the two SSB mechanism; for example this sector couples to the fermions through the Yukawa Lagrangian generating additional constraints.

SSB scheme

As a theory with a larger symmetry groups, the 331 model has a Spontaneous Symmetry Breaking (SSB) scheme that must contain at least two breaking transitions: one that leads from 331 to 321 gauge symmetry groups, and another one from 321 to 31. Considering three complex scalar triplets, we can set the following hierarchical symmetry breaking:

$$\text{SU}(3)_L \times \text{U}(1)_X \xrightarrow[1.T.]{\langle \chi \rangle} \text{SU}(2)_L \times \text{U}(1)_Y \xrightarrow[2.T.]{\langle \rho, \eta \rangle} \text{U}(1)_Q. \quad (3.11)$$

In the first transition (1.T.): we have 5 broken generators due to the scalar field $\langle \chi \rangle$, which are manifested in the mass acquisition of 5 gauge bosons. In the 2.T. due to scalars $\langle \rho, \eta \rangle$, three other generators are broken, giving rise to three gauge bosons and finally one massless gauge boson (the photon). Thus, in addition to the three SM gauge bosons (W^\pm , Z), we obtain 5 new massive weak bosons.

Spectrum

Taking into account the scheme of spontaneous symmetry breaking (SSB) $31 \rightarrow 21 \rightarrow 1$, the most general Higgs potential for any type of 331 model contain three Higgs triplets. In this work we focus in the model with $\beta = 1/\sqrt{3}$, which do not generate exotic charges. In this model in order to give mass to all fermions, it is necessary to introduce three scalar triplets. The usual basis representation are shown in the table 3.2.

Table 3.2: Scalar Spectrum for 331 model with $\beta = \frac{1}{\sqrt{3}}$

scalar triplet	Q_Φ	Y_Φ	X_Φ
$\chi = \begin{pmatrix} \chi^\pm \\ \frac{1}{\sqrt{2}} (\xi_{\chi 0} \pm i\zeta_{\chi 0}) \\ \frac{1}{\sqrt{2}} (\xi_{\chi 3} \pm i\zeta_{\chi 3}) \end{pmatrix}$	$\begin{pmatrix} \pm 1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} \pm \frac{1}{2} \\ \pm \frac{1}{2} \\ 0 \end{pmatrix}$	$\frac{1}{3}$
$\rho = \begin{pmatrix} \rho^\pm \\ \frac{1}{\sqrt{2}} (\xi_{\rho 2} \pm i\zeta_{\rho 2}) \\ \frac{1}{\sqrt{2}} (\xi_{\rho 3} \pm i\zeta_{\rho 3}) \end{pmatrix}$	$\begin{pmatrix} \pm 1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} \pm \frac{1}{2} \\ \pm \frac{1}{2} \\ 0 \end{pmatrix}$	$\frac{1}{3}$
$\eta = \begin{pmatrix} \frac{1}{\sqrt{2}} (\xi_\eta \pm i\zeta_\eta) \\ \eta_2^\mp \\ \eta_3^\mp \end{pmatrix}$	$\begin{pmatrix} 0 \\ \mp 1 \\ \mp 1 \end{pmatrix}$	$\begin{pmatrix} \mp \frac{1}{2} \\ \mp \frac{1}{2} \\ \mp 1 \end{pmatrix}$	$-\frac{2}{3}$

As we can see, the scalar triplets that break symmetry in the second transition, ρ and η , are non identical, since they have different quantum number on X ; defining a unique scalar basis for CP studies. Thus, we must not to propose GCP transformations.

VEV's (Real)

For the 331 model considered here, with $\beta = \frac{1}{\sqrt{3}}$, and using the commutation relation, as we show in the appendix (D), we can find the following values for the vacuum state:

$$\langle \chi \rangle = \begin{pmatrix} 0 \\ 0 \\ \nu_\chi \end{pmatrix}, \quad \langle \rho \rangle = \begin{pmatrix} 0 \\ \nu_{\rho_1} \\ \nu_{\rho_2} \end{pmatrix}, \quad \langle \eta \rangle = \begin{pmatrix} \nu_\eta \\ 0 \\ 0 \end{pmatrix}. \quad (3.12)$$

Since in the second transition there are not solutions with VEV in the first and second component simultaneously in a single triplet, it is necessary to take at least two scalar fields to adjust VEV in all components, so that we can ensure the mass acquisition for all fermions.

3.3 Higgs Lagrangian

The Higgs Lagrangian contains the interactions between the scalar and vector bosons through the kinetic Lagrangian, and scalar self-interactions with the Higgs potential.

3.3.1 Kinetic Terms

The covariant derivative allows us to write the coupling of the gauge bosons with the scalar fields (ϕ). In general the kinetic term has the form:

$$\mathcal{L}_K = (D_\mu \phi)^\dagger (D^\mu \phi), \quad (3.13)$$

where D_μ is the covariant derivative, defined in the equation (3.7). In particular, we have the kinetic term for the three scalar fields, which is written as:

$$\mathcal{L}_K = (D_\mu \chi)^\dagger (D^\mu \chi) + (D_\mu \rho)^\dagger (D^\mu \rho) + (D_\mu \eta)^\dagger (D^\mu \eta). \quad (3.14)$$

3.3.2 Higgs Potential

The self-interactions between scalar bosons is given by the Higgs potential. In order to construct these terms, we have to take into account that they must be hermitian, renormalizable and $SU(3)_L \times U(1)_X$ gauge invariant. $SU(3)$ symmetry can be ensured by constructing product combinations between scalar fields such that they form singlets. The most general Higgs potential considering $\beta = \frac{1}{\sqrt{3}}$ is:

$$\begin{aligned} V_H = & m_1 \chi^\dagger \chi + m_2 \rho^\dagger \rho + m_3 \eta^\dagger \eta + m_4 (\chi^\dagger \rho + h.c.) + f (\epsilon^{ijk} \eta_i \rho_j \chi_k + h.c.) \\ & + l_1 (\chi^\dagger \chi)^2 + l_2 (\rho^\dagger \rho)^2 + l_3 (\eta^\dagger \eta)^2 + l_4 \chi^\dagger \chi \rho^\dagger \rho + l_5 \chi^\dagger \chi \eta^\dagger \eta \\ & + l_6 \rho^\dagger \rho \eta^\dagger \eta + l_7 \chi^\dagger \chi \eta^\dagger \eta + l_8 \chi^\dagger \rho \rho^\dagger \chi + l_9 \eta^\dagger \rho \rho^\dagger \eta \\ & + (l_{10} \chi^\dagger \rho \chi^\dagger \rho + h.c.) + (l_{11} \rho^\dagger \rho \rho^\dagger \chi + h.c.) + (l_{12} \eta^\dagger \eta \chi^\dagger \rho + h.c.) \\ & + (l_{13} \chi^\dagger \chi \chi^\dagger \rho + h.c.) + (l_{14} \eta^\dagger \eta \rho^\dagger \eta + h.c.). \end{aligned} \quad (3.15)$$

3.4 Yukawa Lagrangian

For the proposed model with chiral dependence in SU(3) gauge group and considering the triplets for the scalar fields showing in the table (3.2), the most general Yukawa Lagrangian that couple left and right handed fermions to Higgs fields in a gauge invariant way under SU(3)_L and U(1)_X has the following structure:

Quarks

$$\begin{aligned} \mathcal{L}_Y^Q = & \quad \bar{Q}_L^{(a)} \left(\Gamma_{a,i}^{(D,\rho)} \rho D_R^{(i)} + \Gamma_{a,i}^{(U,\eta)} \eta U_R^{(i)} + \Gamma_{a,a'}^{(J,\chi)} \chi J_R^{(a')} \right) \\ & + \quad \bar{Q}_L^{(3)} \left(\Gamma_{3,i}^{(D,\eta)} \eta^* D_R^{(i)} + \Gamma_{3,i}^{(U,\rho)} \rho^* U_R^{(i)} + \Gamma_{3,3}^{(J,\chi)} \chi^* J_R^{(3)} \right) \\ & + \quad h.c. + \mathcal{L}_{Y,(\beta=1/\sqrt{3})}, \end{aligned} \quad (3.16)$$

where Q^a are the left-handed triplets of the first two quark generations with $a = 1, 2$ while Q^3 is the corresponding one of the third generation. In the same way; $J_R^{a'}$ are the new particles right-handed for the first two families, where latin superscripts: $i, j = 1, 2, 3$ run over all the families, with $D_R^{(1,2,3)} = d_R, s_R, b_R$ and $U_R^{(1,2,3)} = u_R, c_R, t_R$.

As we can see in the general Lagrangian (eq. (3.16)), the new quarks J_R get mass in the first transition from the coupling to the corresponding scalar triplet χ , then those particles will acquire mass in a high energy scale. On the other side there are terms that we can add due to the choice of $\beta = 1/\sqrt{3}$, which correspond to the following terms:

$$\begin{aligned} \mathcal{L}_{Y,(1/\sqrt{3})}^Q = & \quad \bar{Q}_L^{(a)} \left(\Gamma_{a,a'}^{(J,\rho)} \rho J_R^{(a')} + \Gamma_{a,3}^{(J,\eta)} \eta J_R^{(3)} + \Gamma_{a,i}^{(D,\chi)} \chi D_R^{(i)} \right) \\ & + \quad \bar{Q}_L^{(3)} \left(\Gamma_{3,a'}^{(J,\eta)} \eta^* J_R^{(a')} + \Gamma_{3,3}^{(J,\rho)} \rho^* J_R^{(3)} + \Gamma_{3,i}^{(U,\chi)} \chi^* U_R^{(i)} \right) + h.c.. \end{aligned} \quad (3.17)$$

Those terms mix the new heavy quarks $J_{1,2,3}$ in the mass matrix with the triplets ρ and η at the second transition scale. Although the gauge invariance does not forbid those terms, we can restrict them through extra global symmetries.

Leptons

Analogous to the quark sector, for the leptonic part in the Lagrangian, we have the following expression:

$$\mathcal{L}_Y^\ell = \overline{\ell_L^{(i)}} \left(\Gamma_{i,j}^{(\nu,\rho)} \rho \nu_R^{(j)} + \Gamma_{i,j}^{(e,\eta)} \eta e_R^{(j)} + \Gamma_{i,j}^{(L,\chi)} \chi L_R^{(j)} \right) + h.c. + \mathcal{L}_{Y,(\beta=1/\sqrt{3})}^\ell, \quad (3.18)$$

$$\mathcal{L}_{Y,(1/\sqrt{3})}^\ell = \overline{\ell_L^{(i)}} \left(\Gamma_{i,j}^{(L,\rho)} \rho L_R^{(j)} + \Gamma_{i,j}^{(\nu,\chi)} \chi \nu_R^{(j)} \right), \quad (3.19)$$

where the first equation is the general Lagrangian for leptons considering the coupling with right-handed neutrinos, and the second one are the additional terms due to the choice for $\beta = 1/\sqrt{3}$.

CP Violating - 331 model

Implement the CP violation mechanism in the $SU(3)_C \times SU(3)_L \times U(1)_X$ model introduce interesting phenomenology that may be constrained from experimental data for the explicit or spontaneous CP violation. Some CP violation mechanisms in this model have been considered in the literature [18, 19, 20]. In general 331 models require at least three scalar triplets in order to generate the correct particle masses of the model. Considering that each scalar triplet admits at least three VEV, we are going to have at least three phases.

Otherwise, 331 models have two triplets which break local gauge group to $U(1)_Q$ in the second transition. Since these triplets have different quantum numbers, we have a unique basis to analyze explicit or spontaneous CP violation. In this chapter we consider the same structure for 331 model as in the previous chapter with the difference that we are going to introduce complex VEV's and some global symmetries.

4.1 VEV's (Complex)

In the previous chapter real vacuum expectation values were introduced in (eq. (3.12)). Now, we consider the most general complex VEV structure as:

$$\langle \chi \rangle = \begin{pmatrix} 0 \\ 0 \\ |u|e^{i\alpha_x} \end{pmatrix}, \quad \langle \rho \rangle = \begin{pmatrix} 0 \\ |\nu_1|e^{i\alpha_{\rho_1}} \\ |\nu_2|e^{i\alpha_{\rho_2}} \end{pmatrix}, \quad \langle \eta \rangle = \begin{pmatrix} |\omega|e^{i\alpha_\eta} \\ 0 \\ 0 \end{pmatrix}. \quad (4.1)$$

Taking into account the above expressions for the triplets, we have initially 4 phases associated with CP violation phenomenology, however these phases may or may not be physical phases. For that, we must check how the triplets transform under rotations of $SU(3)_L$ and $U(1)_L$ local groups and to study the stationary solutions of the Higgs potential.

4.1.1 SU(3) Rotation

The Higgs potential must be invariant under transformations of the $SU(3)_L$ group. First, we define the transformation over the scalar fields in the form:

$$SU(3) : \quad \phi_a \rightarrow \phi'_a = e^{i\vec{\alpha}_a \cdot \vec{T}} \phi_a , \quad (4.2)$$

where ϕ is one of the three scalar triplets, \vec{T} are the group generators for the $SU(3)_L$ and $\vec{\alpha}$ is the rotation angle. Applying the transformation over the Higgs potential, we obtain the following complex terms:

$$\begin{aligned} V'_H = \dots &+ (m_4 e^{i(\vec{\alpha}_\chi - \vec{\alpha}_\rho) \cdot \vec{T}} \chi'^\dagger \rho' + h.c.) + (f e^{i(\vec{\alpha}_\rho + \vec{\alpha}_\eta + \vec{\alpha}_\chi) \cdot \vec{T}} \epsilon^{ijk} \rho'_i \eta'_j \chi'_k + h.c.) \\ &+ [l_{10} e^{i2(\vec{\alpha}_\chi - \vec{\alpha}_\rho) \cdot \vec{T}} \chi'^\dagger \rho' \chi'^\dagger \rho' + h.c.] + [\rho'^\dagger \rho' (l_{11} e^{-i(\vec{\alpha}_\chi - \vec{\alpha}_\rho) \cdot \vec{T}} \rho'^\dagger \chi' + h.c.)] \\ &+ [\eta'^\dagger \eta' (l_{12} e^{i(\vec{\alpha}_\chi - \vec{\alpha}_\rho) \cdot \vec{T}} \chi'^\dagger \rho' + h.c.)] + [\chi'^\dagger \chi' (l_{13} e^{i(\vec{\alpha}_\chi - \vec{\alpha}_\rho) \cdot \vec{T}} \chi'^\dagger \rho' + h.c.)] \\ &+ [l_{14} e^{-i(\vec{\alpha}_\chi - \vec{\alpha}_\rho) \cdot \vec{T}} \eta'^\dagger \chi' \rho'^\dagger \eta' + h.c.] + \dots \end{aligned}$$

Since the potential is $SU(3)$ invariant, we take $V'_H = V_H$, obtaining the following relations between the rotation angles:

$$\alpha_\chi = \alpha_\rho \quad , \quad \alpha_\eta = -2\alpha_\rho . \quad (4.3)$$

In order to study the transformation of the scalar triplets basis, we need the operator in matricial form¹, using the spectral theorem and the BCH relation for unitary special groups $SU(N)$ [21]. As shown in appendix E, the transformation can be represented as:

$$e^{i\mathbb{M}} = \sum_{m=0}^{\infty} \frac{(-i\mathbb{M})^m}{m!} = \sum_{n=0}^{N-1} f_n(\mathbb{M}) \mathbb{M}^n . \quad (4.4)$$

with \mathbb{M} a hermitian matrix, which in our case is the product between the rotation α with the group generators: $\mathbb{M} = \vec{\alpha}_a \cdot \vec{T}$. This matrix satisfies the Cayley-Hamilton theorem; $M^3 = I \det(M) + \frac{1}{2} M \text{tr}(M^2)$. Applying the spectral decomposition for the special unitary group $SU(3)$, it is $N = 3$, we can apply the general formula for the f_n coefficients, which are given by [22]:

$$e^{i\alpha\mathbb{M}} = \sum_{k=0,1,2} \mathcal{F} \left[\mathbb{M}^2 + \frac{2}{\sqrt{3}} \sin(\phi + 2\pi k/3) \mathbb{M} - \frac{1}{3} (1 + 2 \cos(2\phi + 4\pi k/3)) \mathbb{I} \right] , \quad (4.5)$$

where;

$$\mathcal{F} = \frac{e^{\frac{2}{\sqrt{3}} i\alpha \sin(\phi + 2\pi k/3)}}{1 - 2 \cos(2\phi + 4\pi k/3)} , \quad \text{and}$$

¹Due to $SU(3)$ generators do not satisfy $(\vec{n}_a \cdot \vec{T})^2 = \mathbb{I}$, we can not use the Euler's formula

$$\phi = \frac{1}{3} \left[\arccos \left(\frac{3\sqrt{3}}{2} \det(\mathbb{M}) \right) - \frac{\pi}{2} \right].$$

Making the explicit calculations, as shown in the appendix (E), the following transformation matrix is obtained, where the parameters; $m_{1,2,3}$, $n_{1,2,3}$ and $T_{1,2,3}$ are also defined in the appendix (E).

$$e^{i\vec{\alpha}\cdot\vec{T}} \equiv \mathbb{T} = \frac{1}{2} \begin{pmatrix} 2T_1 & m_1 + n_1 & m_2 + n_2 \\ m_1 - n_1 & 2T_2 & m_3 + n_3 \\ m_2 - n_2 & m_3 - n_3 & 2T_3 \end{pmatrix}. \quad (4.6)$$

From the invariance of the Higgs potential, we can rotate the scalar triplets in order to reduce the phases of the VEV from 4 to 3. In particular we want to rotate ρ in order to leave its VEVs in the second component.

Applying \mathbb{T} to each scalar field, and taking into account the relation between the rotation angles (eq. 4.3), as shown in the appendix (F). We obtain the following final rotation matrix that rotates ρ as was indicated and therefore eliminates one complex phase:

$$\mathbb{T} = \begin{pmatrix} \frac{m_3\nu_1}{\nu_2} & m_1 & -\frac{m_1\nu_1}{\nu_2} \\ 0 & \frac{\nu'}{\nu_1} & 0 \\ 0 & m_3 & -\frac{m_3\nu_1}{\nu_2} \end{pmatrix}. \quad (4.7)$$

Thus, in the vacuum state the scalar triplets (ρ, η, χ) rotates into new basis, in which their components are in general complex values:

$$\begin{aligned} \langle \chi' \rangle &= \begin{pmatrix} 0 \\ 0 \\ u' \end{pmatrix}, \quad \langle \rho' \rangle = \begin{pmatrix} 0 \\ \nu' \\ 0 \end{pmatrix}, \quad \langle \eta' \rangle = \begin{pmatrix} \omega' \\ 0 \\ 0 \end{pmatrix}, \\ u' &= \left(\frac{\nu_2}{\nu_1 m_3} \right)^2 u, \quad \nu' = \left(\frac{\nu_2}{m_3} \right)^2 \frac{1}{\nu_1}, \quad \omega' = 2 \left(\frac{\nu_1 m_3}{\nu_2} \right) \omega. \end{aligned} \quad (4.8)$$

Using this basis; only 3 complex phases are considered in the vacuum state. Furthermore, by applying a $U(1)_X$ phase rotation, we can eliminate another phase, as shown below.

4.1.2 $U(1)$ Rotation

The transformation associated to $U(1)$ group, which operates over the scalar fields ϕ can be defined as;

$$U(1): \quad \phi_a \rightarrow \phi'_a = e^{i\theta_a \cdot X} \phi_a, \quad (4.9)$$

where the fields have a new quantum number (x) associated to the generator X of the $U(1)_X$ group. The generator for $U(1)_X$ is diagonal, so we can consider this transformation as a field

rotation by a phase. It is necessary to take in mind the corresponding “ X ” quantum number for each field. The potential terms that obtain a resultant phase are:

$$\begin{aligned}
V'_H = \dots &+ (m_4 e^{i\frac{1}{3}(\theta_x - \theta_\rho)} \chi'^\dagger \rho' + h.c.) + (f e^{i\frac{1}{3}(\theta_\rho - 2\theta_\eta + \theta_x)} \epsilon^{ijk} \rho'_i \eta'_j \chi'_k + h.c.) \\
&+ [l_{10} e^{i\frac{2}{3}(\theta_x - \theta_\rho)} \chi'^\dagger \rho' \chi'^\dagger \rho' + h.c.] + [\rho'^\dagger \rho' (l_{11} e^{-i\frac{1}{3}(\theta_x - \theta_\rho)} \rho'^\dagger \chi' + h.c.)] \\
&+ [\eta'^\dagger \eta' (l_{12} e^{i\frac{1}{3}(\theta_x - \theta_\rho)} \chi'^\dagger \rho' + h.c.)] + [\chi'^\dagger \chi' (l_{13} e^{i\frac{1}{3}(\theta_x - \theta_\rho)} \chi'^\dagger \rho' + h.c.)] \\
&+ [l_{14} e^{-i\frac{1}{3}(\theta_x - \theta_\rho)} \eta'^\dagger \chi' \rho'^\dagger \eta' + h.c.] + \dots .
\end{aligned}$$

From the invariance of the potential, we have the relation for the angles:

$$\theta_\rho = \theta_\eta = \theta_x = \theta . \quad (4.10)$$

After the SU(3) rotation, the VEVs (eq. 4.8) are:

$$\langle \chi \rangle = \begin{pmatrix} 0 \\ 0 \\ |u| e^{i\alpha_\chi} \end{pmatrix} , \quad \langle \rho \rangle = \begin{pmatrix} 0 \\ |\nu| e^{i\alpha_\rho} \\ 0 \end{pmatrix} , \quad \langle \eta \rangle = \begin{pmatrix} |\omega| e^{i\alpha_\eta} \\ 0 \\ 0 \end{pmatrix} . \quad (4.11)$$

Applying U(1) rotation, with $\theta = \frac{3}{2}\alpha_\chi$ we can eliminate the phase associated to the scalar field χ to avoid CP violation in the first transition. Thus, we obtain the following VEV structures:

$$\langle \chi' \rangle = \begin{pmatrix} 0 \\ 0 \\ |u| \end{pmatrix} , \quad \langle \rho' \rangle = \begin{pmatrix} 0 \\ |\nu| e^{i\delta_\rho} \\ 0 \end{pmatrix} , \quad \langle \eta' \rangle = \begin{pmatrix} |\omega| e^{i\delta_\eta} \\ 0 \\ 0 \end{pmatrix} \quad (4.12)$$

where the phases are redefined as:

$$\delta_\rho = \alpha_\rho + \frac{1}{2}\alpha_\chi , \quad \delta_\eta = \alpha_\eta + \frac{1}{2}\alpha_\chi . \quad (4.13)$$

For our purpose, we are going to consider in the next chapter the cartesian form for the above scalar triplets in the vacuum state, it is:

$$\langle \chi \rangle = \begin{pmatrix} 0 \\ 0 \\ u \end{pmatrix} , \quad \langle \rho \rangle = \begin{pmatrix} 0 \\ v_1 + iv_2 \\ 0 \end{pmatrix} , \quad \langle \eta \rangle = \begin{pmatrix} w_1 + iw_2 \\ 0 \\ 0 \end{pmatrix} . \quad (4.14)$$

Thus, we obtain only two possible phases as candidates to produce CP-violating interactions. However, as we will see below, from the stationary conditions of the Higgs potential, these phases are not mutually independent.

4.2 Higgs Potential with U(1) Global Symmetry

Taking the full Higgs potential (eq. (3.15)) we can see that it can be separated in two parts; the first one with real terms whose coefficients are also real and the other one is the complex part, whose coefficients in general are complex along with their corresponding hermitian conjugate terms:

$$V_H = V_{\mathbb{R}} + V_{\mathbb{C}} ,$$

where:

$$\begin{aligned} V_{\mathbb{R}} = & \left[\begin{aligned} & m_1 \chi^\dagger \chi + m_2 \rho^\dagger \rho + m_3 \eta^\dagger \eta \\ & + l_1 (\chi^\dagger \chi)^2 + l_2 (\rho^\dagger \rho)^2 + l_3 (\eta^\dagger \eta)^2 + l_4 \chi^\dagger \chi \rho^\dagger \rho + l_5 \chi^\dagger \chi \eta^\dagger \eta \\ & + l_6 \rho^\dagger \rho \eta^\dagger \eta + l_7 \chi^\dagger \eta \eta^\dagger \chi + l_8 \chi^\dagger \rho \rho^\dagger \chi + l_9 \eta^\dagger \rho \rho^\dagger \eta \end{aligned} \right] \end{aligned} \quad (4.15)$$

$$\begin{aligned} V_{\mathbb{C}} = & \left[\begin{aligned} & m_4 \chi^\dagger \rho + f \epsilon^{ijk} \eta_i \rho_j \chi_k + l_{10} \chi^\dagger \rho \chi^\dagger \rho \\ & + l_{11} \rho^\dagger \rho \rho^\dagger \chi + l_{12} \eta^\dagger \eta \chi^\dagger \rho \\ & + l_{13} \chi^\dagger \chi \chi^\dagger \rho + l_{14} \eta^\dagger \chi \rho^\dagger \eta + \mathbf{h.c.} \end{aligned} \right] . \end{aligned} \quad (4.16)$$

The complex sector can be restricted if we demand additional global symmetries. For example, we can introduce a U(1) symmetry defined as:

$$\chi \rightarrow \chi , \quad \rho \rightarrow e^{i\Theta} \rho , \quad \eta \rightarrow e^{-i\Theta} \eta , \quad (4.17)$$

where χ stays invariant, while the other two scalar fields, ρ and η , change by a opposite phase. Under this transformation, the real potential stays invariant and only one term survives into the complex potential:

$$V_H = V_{\mathbb{R}} + \left(f \epsilon^{ijk} \eta_i \rho_j \chi_k + h.c. \right) . \quad (4.18)$$

4.2.1 Explicit CP invariance

Considering the scalar fields in vacuum state as real $\{\langle \chi \rangle, \langle \rho \rangle, \langle \eta \rangle\} \in \mathbb{R}$, the unique CP phase must come from the potential parameters. However, we can define a base rotation for each scalar field, that absorb this phase. For instance, if we set:

$$\chi \rightarrow \chi' = e^{i\theta_\chi} \chi \implies \chi = e^{-i\theta_\chi} \chi' , \quad (4.19)$$

$$\rho \rightarrow \rho' = e^{i\theta_\rho} \rho \implies \rho = e^{-i\theta_\rho} \rho' , \quad (4.20)$$

$$\eta \rightarrow \eta' = e^{i\theta_\eta} \eta \implies \eta = e^{-i\theta_\eta} \eta' , \quad (4.21)$$

after replacing into the potential (4.18), we can obtain the following expression:

$$V_H = V_{\mathbb{R}} + \left(|f| e^{i\alpha_f} \epsilon^{ijk} e^{i(\theta_\eta + \theta_\rho + \theta_\chi)} \eta'_i \rho'_j \chi'_k + h.c. \right). \quad (4.22)$$

As a consequence, any phase choice that satisfy $\theta_\eta + \theta_\rho + \theta_\chi = -\alpha_f$, leaves the potential completely real, and therefore explicitly CP invariant. For example we can take:

$$\theta_\chi = -\frac{1}{3}\alpha_f, \quad \theta_\rho = -\frac{1}{3}\alpha_f, \quad \theta_\eta = -\frac{1}{3}\alpha_f.$$

4.2.2 Spontaneous CP invariance

In the previous section we considered the scalar fields as real, now we must assume the case in which CP phases comes from the vacuum state. After a $SU(3)_L$ and $U(1)_X$ transformation we obtained $\{\langle\rho\rangle, \langle\eta\rangle\} \in \mathbb{C}$ and $\langle\chi\rangle \in \mathbb{R}$, shown in equation (4.12).

$$\langle\chi\rangle = \begin{pmatrix} 0 \\ 0 \\ |u| \end{pmatrix}, \quad \langle\rho\rangle = \begin{pmatrix} 0 \\ |\nu| e^{i\delta_\rho} \\ 0 \end{pmatrix}, \quad \langle\eta\rangle = \begin{pmatrix} |\omega| e^{i\delta_\eta} \\ 0 \\ 0 \end{pmatrix}. \quad (4.23)$$

After replacing these into the potential evaluated at the vacuum state; the real part is maintained invariant, but the complex term contains the phases as:

$$\langle V_H \rangle = \langle V_{\mathbb{R}} \rangle - \left(f u |\nu| |\omega| e^{-i(\delta_\eta + \delta_\rho)} + f^* u |\nu| |\omega| e^{i(\delta_\eta + \delta_\rho)} \right), \quad (4.24)$$

where the minus sign comes from the anti-symmetrical property of the Levi-Civita tensor. On the other hand, we must minimize the potential to find the tadpole relations for the phases, so we have:

$$\frac{\partial \langle V_H \rangle}{\partial \delta_{\rho, \eta}} = 0, \quad \text{where; } \frac{\partial \langle V_{\mathbb{R}} \rangle}{\partial \delta_{\rho, \eta}} \overset{0}{\nearrow}.$$

Minimizing, we have the following expression:

$$i f u |\nu| |\omega| e^{-i(\delta_\eta + \delta_\rho)} (\delta_\rho + \delta_\eta) - i f^* u |\nu| |\omega| e^{i(\delta_\eta + \delta_\rho)} (\delta_\rho + \delta_\eta) = 0$$

$$\begin{aligned} \cancel{i u |\nu| |\omega| f e^{-i(\delta_\eta + \delta_\rho)} (\delta_\rho + \delta_\eta)} &= \cancel{i u |\nu| |\omega| f^* e^{i(\delta_\eta + \delta_\rho)} (\delta_\rho + \delta_\eta)} \\ f e^{-i(\delta_\eta + \delta_\rho)} &= f^* e^{i(\delta_\eta + \delta_\rho)} \\ e^{i2\alpha_f} = \frac{f}{f^*} &= e^{i2(\delta_\eta + \delta_\rho)} \\ \alpha_f &= \delta_\eta + \delta_\rho. \end{aligned} \quad (4.25)$$

So we have a relation between the phases, leaving one independent phase for the model (i.e. $\delta_\eta = \alpha_f - \delta_\rho$). Making again a rotation of the base over the scalar field, we can cancel the phases from the vacuum state as:

$$\chi \rightarrow \chi' = e^{i\theta_\chi} \chi \quad ; \quad \theta_\chi = 0 , \quad (4.26)$$

$$\rho \rightarrow \rho' = e^{i\theta_\rho} \rho \quad ; \quad \theta_\rho = -\delta_\rho , \quad (4.27)$$

$$\eta \rightarrow \eta' = e^{i\theta_\eta} \eta \quad ; \quad \theta_\eta = -\delta_\eta = \delta_\rho - \alpha_f . \quad (4.28)$$

After replacing into the potential (eq. (4.18)), we find:

$$\begin{aligned} V'_H &= V_{\mathbb{R}} + \left(|f| e^{i\alpha_f} \epsilon^{ijk} e^{-i(\delta_\rho - \delta_\rho + \alpha_f)} \eta'_i \rho'_j \chi'_k + h.c. \right) \\ &= V_{\mathbb{R}} + |f| \left(\epsilon^{ijk} \eta'_i \rho'_j \chi'_k + \epsilon_{ijk} \eta'^i \rho'^j \chi'^k \right) . \end{aligned} \quad (4.29)$$

This potential is in a real space, so under the rephasing transformation we eliminate simultaneously the phases from the VEV and from the complex parameter f , obtaining a CP invariant potential, even if the fields originally had complex vacuum expected value.

4.3 Breaking of Symmetry $U(1) \rightarrow Z_2$

As we can see in the previous section, the $U(1)$ global symmetry leaves the model CP invariant in the scalar sector, even if we consider complex VEV or complex parameters. On the other hand, we can introduce a symmetry breaking to obtain a discrete symmetry as (Z_2). This symmetry allows terms that can violate CP symmetry, in particular these new terms could generate an irreducible phase, in that way this breaking is equivalent to make a soft breaking of the global symmetry.

For example, we can consider $\Theta = \pi$ in the transformation from equation (4.17). This choice is equivalent to consider a Z_2 symmetry. Thus this transformation can be defined as:

$$\chi \rightarrow \chi, \quad \eta \rightarrow -\eta, \quad \rho \rightarrow -\rho . \quad (4.30)$$

Taking again the Higgs potential as real part and complex part, we can find that real part is invariant but in the complex terms we have an additional term as show in the following expression:

$$V_H = V_{\mathbb{R}} + \left(f \epsilon^{ijk} \eta_i \rho_j \chi_k + l_{10} \chi^\dagger \rho \chi^\dagger \rho + h.c. \right) . \quad (4.31)$$

Analyzing this potential under field rotations to search invariance, we have to consider again two cases; explicit and spontaneous phases.

4.3.1 Explicit CP invariance

First, we assume that the scalar fields are real $\{\langle\chi\rangle, \langle\rho\rangle, \langle\eta\rangle\} \in \mathbb{R}$ and we propose the same basis rotation in equations as (4.19) - (4.21), so after replacing into the potential (eq. 4.31), we have:

$$V_H = V_{\mathbb{R}} + \left(|f| e^{i\alpha_f} \epsilon^{ijk} e^{i(\theta_\eta + \theta_\rho + \theta_\chi)} \eta'_i \rho'_j \chi'_k + |l_{10}| e^{i\alpha_{10}} e^{2i(\theta_\chi - \theta_\rho)} \chi'^\dagger \rho' \chi'^\dagger \rho' + h.c. \right). \quad (4.32)$$

CP invariance for this potential is manifested if the following relations are satisfied:

$$\theta_\eta + \theta_\rho + \theta_\chi = -\alpha_f, \quad (4.33)$$

$$2(\theta_\chi - \theta_\rho) = -\alpha_{10}. \quad (4.34)$$

Again, we have the freedom to choose the (θ) angles, in a way that the above equations are satisfied, for example:

$$\theta_\chi = -\frac{\alpha_f}{2}, \quad \theta_\eta = -\frac{\alpha_{10}}{2}, \quad \theta_\rho = \frac{\alpha_{10} - \alpha_f}{2}.$$

Introducing that in the potential (4.32), we have:

$$V'_H = V_{\mathbb{R}} + |f| \left(\epsilon^{ijk} \eta'_i \rho'_j \chi'_k + h.c. \right) + |l_{10}| \left(\chi'^\dagger \rho' \chi'^\dagger \rho' + h.c. \right). \quad (4.35)$$

In that way we have the whole potential in a real space, **thus we obtain CP symmetry conservation.**

4.3.2 Spontaneous CP Violation

In the same way that was proposed in section 4.2.2, we consider the vacuum expectation values as complex. After $SU(3)_L$ and $U(1)_X$ rotation we have a spectrum with two complex phases, where the scalar field χ in the ground state is real:

$$\{\langle\rho\rangle, \langle\eta\rangle, \langle\chi\rangle\} \in \mathbb{C} \xrightarrow{U(1)_X} \{\langle\rho\rangle, \langle\eta\rangle\} \in \mathbb{C}, \quad \langle\chi\rangle \in \mathbb{R}$$

Taking the VEVs as in equation (4.12), we obtain the Higgs potential in the vacuum state. Again we find that the real part remains invariant, while the complex part takes the form:

$$\langle V_H \rangle = \langle V_{\mathbb{R}} \rangle - \left(f u |\nu| |\omega| e^{-i(\delta_\eta + \delta_\rho)} + f^* u |\nu| |\omega| e^{i(\delta_\eta + \delta_\rho)} \right). \quad (4.36)$$

Minimizing the potential and following the same procedure as section 4.2.2, we find the relationship between the phases, leaving just one physical phase associate with CP symmetry violation:

$$\delta_\eta = \alpha_f - \delta_\rho.$$

Now, we can use in this case the base rotation to cancel all the phases in the ground state, so we have again:

$$\chi \rightarrow \chi' = e^{i\theta_\chi} \chi \quad ; \quad \theta_\chi = 0 , \quad (4.37)$$

$$\rho \rightarrow \rho' = e^{i\theta_\rho} \rho \quad ; \quad \theta_\rho = -\delta_\rho , \quad (4.38)$$

$$\eta \rightarrow \eta' = e^{i\theta_\eta} \eta \quad ; \quad \theta_\eta = -\delta_\eta = \delta_\rho - \alpha_f \quad (4.39)$$

Using the inverse transformation to obtain the fields as function of the new fields, and replacing into the potential (eq. (4.31)), we can find:

$$\begin{aligned} V_H &= V_{\mathbb{R}} + \left(|f| e^{i\alpha_f} e^{-i(\delta_\rho - \delta_\rho + \alpha_f)} \epsilon^{ijk} \eta'_i \rho'_j \chi'_k + h.c. \right) \\ &+ \left(|l_{10}| e^{i\alpha_{10}} e^{i2\delta_\rho} \chi'^\dagger \rho' \chi'^\dagger \rho' + h.c. \right) , \end{aligned} \quad (4.40)$$

computing the phases, we finally obtain:

$$V_H = V_{\mathbb{R}} + |f| \left(\epsilon^{ijk} \eta'_i \rho'_j \chi'_k + h.c. \right) + \left(|l_{10}| e^{i\sigma} \chi'^\dagger \rho' \chi'^\dagger \rho' + h.c. \right) . \quad (4.41)$$

As a result, the parameter in the second term become real but the third term can not reduce the phase, leaving a new one phase that obey: $\sigma = \alpha_{10} + 2\delta_\rho$, so we have one remaining phase considering the rotation basis and the VEVs as complex, thus **the potential in general is non invariant under CP transformation**².

In the following chapters we are going to consider this case of CP violation, imposing the discrete symmetry Z_2 for the potential that allows the “ l_{10} ” term.

²There is a very particular case in which ($\alpha_{10} = -2\delta_\rho$), then CP is conserving

Mass states in CPV 331 model

In the above chapter, we found that a 331 model with a Z_2 symmetry admits spontaneous CP violation in the scalar sector. In this chapter, we are going to analyze the minimal conditions and the mass eigenstates to obtain all the scalar spectrum and CP violation consequences.

5.1 Physical Spectrum - Scalar Sector

In the table 3.2, we can define the scalar fields in the electroweak basis. Thus, we must rotate the basis in order to obtain the physical states for the scalar fields and identify the Goldstone bosons and the Higgs massive fields. Taking the potential with symmetry Z_2 (eq. (4.31)):

$$\begin{aligned}
 V_{H,(Z_2)} = & \quad m_1\chi^\dagger\chi + m_2\rho^\dagger\rho + m_3\eta^\dagger\eta \\
 & + l_1(\chi^\dagger\chi)^2 + l_2(\rho^\dagger\rho)^2 + l_3(\eta^\dagger\eta)^2 + l_4\chi^\dagger\chi\rho^\dagger\rho + l_5\chi^\dagger\chi\eta^\dagger\eta \\
 & + l_6\rho^\dagger\rho\eta^\dagger\eta + l_7\chi^\dagger\eta\eta^\dagger\chi + l_8\chi^\dagger\rho\rho^\dagger\chi + l_9\eta^\dagger\rho\rho^\dagger\eta \\
 & + \left[f\epsilon^{ijk}\eta_i\rho_j\chi_k + l_{10}\chi^\dagger\rho\chi^\dagger\rho + \mathbf{h.c.} \right].
 \end{aligned} \tag{5.1}$$

Evaluating the scalar triplets in the groundstate (eq. (4.14) including a factor; $\frac{1}{\sqrt{2}}$) and replacing into the above potential (eq. (5.1)), we proceed to minimize the potential with respect to the vacuum expectation values, it is:

$$\frac{\partial\langle V_H \rangle}{\partial\nu_i} = 0, \quad \nu_i = \nu_1, \nu_2, w_1, w_2, u \quad ,$$

where $\langle V_H \rangle$ is the Higgs potential evaluated in the vacuum. After minimization, we can find the minimal conditions or the tadpoles for this potential that we can written as:

$$m_1 = -2l_1u^2 - l_4v_1^2 - l_4v_2^2 - l_5w_1^2 - l_5w_2^2 + \frac{fv_1w_1}{u} - \frac{fv_2w_2}{u}, \quad (5.2)$$

$$m_2 = -l_4u^2 - 2l_2v_1^2 - 2l_2v_2^2 - l_6w_1^2 - l_6w_2^2 + \frac{fuv_1}{2v_1} - \frac{fuv_2}{2v_2}, \quad (5.3)$$

$$m_3 = -l_5u^2 - l_6v_1^2 - l_6v_2^2 - 2l_3w_1^2 - 2l_3w_2^2 + \frac{fuv_1}{2w_1} - \frac{fuv_2}{2w_2}, \quad (5.4)$$

$$w_2 = -w_1 \frac{\nu_2}{\nu_1}. \quad (5.5)$$

Due to the minimal conditions for the potential, there is just one phase related with CP Violation, as was obtained in the section 4.3.2. This occurs because the vacuum expectation phase associated to “ η ” is proportional to the vacuum phase associated with the scalar “ ρ ”, which is equivalent to expression (5.5). In their cartesian form.

Now, replacing the tadpoles (eqs. (5.2)-(5.4)) into the potential (5.1), it is possible to find the mass matrix. For that, the scalar triplets must be shifted as the sum of fields in vacuum state plus the excited states: $\hat{\Phi} = \langle \Phi \rangle + \hat{\Phi}$, as shown in the following expressions:

$$\chi = \begin{pmatrix} \chi^\pm \\ \frac{1}{\sqrt{2}} (\xi_{\chi_0} \pm i\zeta_{\chi_0}) \\ \frac{1}{\sqrt{2}} (\xi_{\chi_3} + u \pm i\zeta_{\chi_3}) \end{pmatrix}, \quad \rho = \begin{pmatrix} \rho^\pm \\ \frac{1}{\sqrt{2}} ((\xi_{\rho_2} + v_1) \pm i(\zeta_{\rho_2} + v_2)) \\ \frac{1}{\sqrt{2}} (\xi_{\rho_3} \pm i\zeta_{\rho_3}) \end{pmatrix}, \quad \eta = \begin{pmatrix} \frac{1}{\sqrt{2}} ((\xi_\eta + w_1) \pm i(\zeta_\eta + w_2)) \\ \eta_2^\mp \\ \eta_3^\mp \end{pmatrix}. \quad (5.6)$$

Then we can obtain the mass matrix terms, for both; the neutral sector and the charged sector. Since there are not mixing terms in the mass parameters between the charged and neutral sectors, due to the electric charge conservation, we can put them into two separated blocks:

$$M^2 = \begin{pmatrix} M_N^2 & 0 \\ 0 & M_C^2 \end{pmatrix},$$

where M_N^2 is a 10×10 matrix for the neutral sector and the matrix M_C^2 is a 4×4 matrix with charged fields.

5.1.1 Charged Sector

In order to obtain the mass matrices for the charged sector we take the second derivative of the potential (4.31) (after replacing the tadpoles) with respect to each charged field, $\Phi_{C_i} = \chi^\pm, \rho^\pm, \eta_2^\mp, \eta_3^\mp$:

$$M_{\chi^\pm \Phi_C}^2 = \left. \frac{\partial^2 V_H}{\partial \chi^\pm \partial \Phi_{C_i}} \right|_{\Phi_{C_i}=0}, \quad M_{\rho^\pm \Phi_C}^2 = \left. \frac{\partial^2 V_H}{\partial \rho^\pm \partial \Phi_{C_i}} \right|_{\Phi_{C_i}=0}, \quad (5.7)$$

$$M_{\eta_2^\mp \Phi_C}^2 = \left. \frac{\partial^2 V_H}{\partial \eta_2^\mp \partial \Phi_{C_i}} \right|_{\Phi_{C_i}=0}, \quad M_{\eta_3^\mp \Phi_C}^2 = \left. \frac{\partial^2 V_H}{\partial \eta_3^\mp \partial \Phi_{C_i}} \right|_{\Phi_{C_i}=0}. \quad (5.8)$$

Finding the mass terms in the basis χ^\pm , η_3^\pm , ρ^\pm , η_2^\pm we obtain a block diagonal matrix with two 2×2 submatrices, (see appendix G.1). These matrices can be diagonalized in a usual way, obtaining the following eigenvalues:

$$\lambda_1 = 0, \quad (5.9)$$

$$\lambda_2 = 0, \quad (5.10)$$

$$\begin{aligned} \lambda_3 &= \frac{1}{2} \left(l_7 + \sqrt{2} f \frac{v_1}{u w_1} \right) \left(u^2 + \frac{w_1^2}{v_1^2} (v_1^2 + v_2^2) \right) \\ &\approx \frac{1}{2} l_7 u^2, \end{aligned} \quad (5.11)$$

$$\begin{aligned} \lambda_4 &= \frac{1}{2} \left(l_9 (v_1^2 + v_2^2) + \sqrt{2} f \frac{u v_1}{w_1} \right) \left(1 + \frac{w_1^2}{v_1^2} \right) \\ &\approx \frac{f}{\sqrt{2}} \left(\frac{u (v_1^2 + w_1^2)}{v_1 w_1} \right), \end{aligned} \quad (5.12)$$

where the approximation to dominant terms was taken $\sim u^2$ or $\sim u$, due to the large energy scale of the first transition. The associated mass eigenstates to these eigenvalues are presented together with the rotation matrices in the section 5.1.3. The null eigenvalues $\lambda = 0$ will be associated with the Goldstone bosons.

5.1.2 Neutral Sector

As in the previous section, we obtain the mass matrices for the neutral sector finding the second derivative of the potential (4.31):

$$M_{\xi_i \Phi_{N_i}}^2 = \left. \frac{\partial^2 V_H}{\partial \xi_i \partial \Phi_{N_i}} \right|_{\Phi_i=0}, \quad M_{\zeta_i \Phi_{N_i}}^2 = \left. \frac{\partial^2 V_H}{\partial \zeta_i \partial \Phi_{N_i}} \right|_{\Phi_i=0}, \quad (5.13)$$

where was separated the neutral fields in its real and imaginary parts, it is $\Phi_N = \zeta_i + \xi_j$, thus we have: $\zeta_i = \zeta_{\chi_2}, \zeta_{\chi_3}, \zeta_{\rho_2}, \zeta_{\rho_3}, \zeta_\eta$ and $\xi_i = \xi_{\chi_2}, \xi_{\chi_3}, \xi_{\rho_2}, \xi_{\rho_3}, \xi_\eta$. Unlike the 331 CP conservative model, in this case we have mixing terms between the real fields and the imaginary fields.

In the basis: $\xi_{\chi_2}, \zeta_{\chi_2}, \xi_{\rho_3}, \zeta_{\rho_3}, \zeta_{\chi_3}, \zeta_{\rho_2}, \zeta_{\eta}, \xi_{\rho_2}, \xi_{\eta}, \xi_{\chi_3}$, it is possible to obtain a block diagonal matrix, as:

$$M_N^2 = \begin{pmatrix} \mathbb{M}_a^2 & 0 \\ 0 & \mathbb{M}_b^2 \end{pmatrix}, \quad (5.14)$$

where \mathbb{M}_a^2 is a 4×4 matrix which could be diagonalized as shown in the appendix H.1, where the following eigenvalues are obtained:

$$\mathbb{M}_{a1}^2(\xi_{\chi_2}, \zeta_{\chi_2}) : \begin{cases} \lambda_5 = 0 \\ \lambda_6 = 0 \end{cases} \quad (5.15a)$$

$$(5.15b)$$

$$\mathbb{M}_{a2}^2(\xi_{\rho_3}, \zeta_{\rho_3}) : \begin{cases} \lambda_7 = \frac{u^2}{2}(l_8 + 2l_{10}) + \frac{f u w_1}{\sqrt{2} v_1} \\ \lambda_8 = \frac{u^2}{2}(l_8 - 2l_{10}) + \frac{f u w_1}{\sqrt{2} v_1} \end{cases} \quad (5.16a)$$

$$(5.16b)$$

Taking again only the dominant term, we obtain for the above eigenvalues that:

$$\lambda_7 \approx \frac{u^2}{2}(l_8 + 2l_{10}) \quad , \quad \lambda_8 \approx \frac{u^2}{2}(l_8 - 2l_{10}). \quad (6.14)$$

In the case of the submatrix \mathbb{M}_b^2 , some considerations are made on the hierarchy of their components. The details of the approximations and the diagonalization process are shown in the appendix H.2. First, two fields can be directly decoupled into one massless and one massive particle:

$$\lambda_{\zeta_{\chi_3}} = 0, \quad (5.17)$$

$$\begin{aligned} \lambda_{\xi_{\chi_3}} &= 2l_1 u^2 + \frac{f w_1}{\sqrt{2} u v_1} (v_1^2 + v_2^2) \\ &\approx 2l_1 u^2. \end{aligned} \quad (5.18)$$

Second, from the remainder terms, we form a singular submatrix of order (4×4) , obtaining:

$$\mathbb{M}_{b2}^2(\zeta_{\rho_2}, \zeta_{\eta}, \xi_{\rho_2}, \xi_{\eta}) : \begin{cases} \lambda_9 = 0 & (5.19a) \\ \lambda_{10} = \frac{f}{\sqrt{2}} \left(\frac{u(v_1^2 + w_1^2)}{v_1 w_1} \right) & (5.19b) \\ \lambda_{11} \approx \frac{1}{2} \left(\frac{l_c w_1^4 - 2l_b w_1^2 v_1^2 - l_a v_1^4}{v_1^2 + w_1^2} \right) \frac{v_1^2 + v_2^2}{v_1^2} & (5.19c) \\ \lambda_{12} \approx \frac{f}{\sqrt{2}} \left(\frac{u(v_1^2 + w_1^2)}{v_1 w_1} \right) & (5.19d) \end{cases}$$

where:

$$l_a = l_4^2 - 4l_1l_2 \quad , \quad -l_c = l_5^2 - 4l_1l_3 \quad , \quad l_b = l_4l_5 - 2l_1l_6.$$

As a result, for the neutral sector, we finally obtained 4 Goldstone bosons, associated to $\lambda_{5,6,9,\zeta_{\chi_3}}$, one light boson at the electroweak scale, associated to the observed Higgs boson (λ_{11}), and five heavy Higgs bosons at the scale of the first transition ($\lambda_{7,8,10,12,\chi_3}$).

5.1.3 Physical States

Using above eigenvalues, we can find the rotation matrix which is constructed from the eigenvectors. We must to check in mind that those eigenvectors must be orthonormal, as it is shown in the appendix G.2.

Charged sector

Considering that this sector has a block diagonal matrix; we can obtain 2 rotation matrices \mathbb{R}_1 y \mathbb{R}_2 associated to the 2 submatrices:

$$\mathbb{R}_1 \begin{pmatrix} \chi^\pm \\ \eta_3^\pm \end{pmatrix} = \begin{pmatrix} G_1^\pm \\ H_1^\pm \end{pmatrix} \quad , \quad \mathbb{R}_1 = \begin{pmatrix} \cos \alpha & e^{-i\delta} \sin \alpha \\ -e^{i\delta} \sin \alpha & \cos \alpha \end{pmatrix} \quad , \quad (5.20)$$

$$\mathbb{R}_2 \begin{pmatrix} \rho^\pm \\ \eta_2^\pm \end{pmatrix} = \begin{pmatrix} G_2^\pm \\ H_2^\pm \end{pmatrix} \quad , \quad \mathbb{R}_2 = \begin{pmatrix} \cos \theta & \sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} . \quad (5.21)$$

These matrices are expressed as function of rotation angles which are defined as:

$$\sin \alpha = -\frac{|w|}{u} \quad ; \quad |w| = \frac{w_1}{v_1} |\nu| \quad , \quad (5.22)$$

$$e^{i\delta} = \frac{v_1 + iv_2}{|\nu|} \quad ; \quad |\nu| = \sqrt{v_1^2 + v_2^2} \quad , \quad (5.23)$$

$$\sin \theta = -\frac{|w|}{\nu_{ew}} \quad ; \quad \nu_{ew} = \sqrt{|w|^2 + |v|^2} . \quad (5.24)$$

Remembering that $|w|$, $|u|$, $|\nu|$ are the norm of the VEV's of each scalar triplets, while ν_{ew} can be defined as the norm of the vacuum in the electroweak sector ρ and η , and where $\nu_{ew} \approx 246$ GeV [23, 24].

After the basis rotation, we can obtain the mass eigenstates with the following consequences: 4 charged Goldstone bosons $G_{1,2}^\pm$ which will give mass to 4 charged Gauge bosons. We also have 4 heavy charged Higgs bosons $H_{1,2}^\pm$, with degenerated states¹. The physical spectrum associated to the charged fields are shown in the table 5.1.

¹The field with opposite charge have the same mass

Table 5.1: Physical Scalar Spectrum

Charged Scalar	Squared Mass
$G_{1,2}^\pm$	0
$G_{1,2,3,4}^0$	0
H_1^\pm	$\frac{1}{2}l_7 u^2$
H_2^\pm, H_4^0, H_5^0	$\frac{f}{\sqrt{2}}u \left(\frac{\nu_{ew}^2}{ v w } \right)$
H_1^0	$\frac{1}{2}(l_8 + l_{10}) u^2$
H_2^0	$\frac{1}{2}(l_8 - l_{10}) u^2$
H_3^0	$2l_1 u^2$
h^0	$\frac{1}{2} \left(\frac{l_c w ^4 - 2l_b w ^2 v ^2 - l_a v ^4}{\nu_{ew}^2} \right)$

Neutral sector

Considering the eigenvalues obtained for the neutral sector (equations, (5.15a),(5.15b),(6.14)), we can obtain the rotation matrix associate to the matrix \mathbb{M}_a^2 , we have:

$$\mathbb{R}_3 \begin{pmatrix} \xi_{\chi_2^0} \\ \zeta_{\chi_2^0} \\ \xi_{\rho_3^0} \\ \zeta_{\rho_3^0} \end{pmatrix} = \begin{pmatrix} G_1^0 \\ G_2^0 \\ H_1^0 \\ H_2^0 \end{pmatrix}, \quad \mathbb{R}_3 = \begin{pmatrix} C_\Omega & 0 & S_\Omega C_\delta & S_\Omega S_\delta \\ 0 & C_\Omega & S_\Omega S_\delta & -S_\Omega C_\delta \\ -S_\Omega C_\delta & -S_\Omega S_\delta & C_\Omega & 0 \\ -S_\Omega S_\delta & S_\Omega C_\delta & 0 & C_\Omega \end{pmatrix}, \quad (5.25)$$

where the angles satisfy the following relations:

$$S_\Omega \equiv \sin \Omega = -\frac{|v|}{u}, \quad S_\delta \equiv \sin \delta = \frac{v_2}{|v|}. \quad (5.26)$$

From the submatrix \mathbb{M}_b^2 we had directly: one neutral Goldstone boson and one neutral Higgs boson. These bosons are:

$$\zeta_{\chi_3^0} = G_3^0, \quad M_{G_3^0}^2 \approx 0, \quad (5.27)$$

$$\xi_{\chi_3^0} = H_3^0, \quad M_{H_3^0}^2 \approx 2l_1 u^2. \quad (5.28)$$

Finally, the eigenvalues for the singular submatrix \mathbb{M}_{b2}^2 . The obtained rotation matrix is in the form:

$$\mathbb{R}_4 \begin{pmatrix} \zeta_{\rho_2^0} \\ \zeta_{\eta^0} \\ \xi_{\rho_2^0} \\ \xi_{\eta^0} \end{pmatrix} = \begin{pmatrix} G_4^0 \\ H_4^0 \\ H_5^0 \\ h^0 \end{pmatrix}, \quad \mathbb{R}_4 = \begin{pmatrix} C_\theta C_\delta & -S_\theta C_\delta & -C_\theta S_\delta & -S_\theta S_\delta \\ -S_\theta C_\delta & -C_\theta C_\delta & S_\theta S_\delta & -C_\theta S_\delta \\ S_\theta S_\delta & C_\theta S_\delta & S_\theta C_\delta & -C_\theta C_\delta \\ C_\theta S_\delta & -S_\theta S_\delta & -C_\theta C_\delta & -S_\theta C_\delta \end{pmatrix}, \quad (5.29)$$

where the relations for δ y θ are giving in the equations (5.23) and (5.24) respectively. In this rotation we obtain one additional Goldstone, other 2 massive Higgs bosons and one Higgs boson at the electroweak scale which corresponds to the SM Higgs boson. A summary of the physical spectrum for the entire scalar sector is shown in the table 5.1.

5.2 Physical Spectrum - Vector Sector

For the mass terms in the vector sector we have the same structure as the CP conservative case. The interactions with the scalar matter are incorporated through the kinetic Higgs Lagrangian. This Lagrangian was introduced in section 3.3.1 in equations (3.13) and (3.14). We can obtain an explicit kinetic Lagrangian and then we can obtain the mass matrices through:

$$\begin{aligned} M_{U^\pm\Phi_i}^2 &= \left. \frac{\partial^2 \mathcal{L}_K}{\partial U^\pm \partial \Phi_i} \right|_{\Phi_i=0}, & M_{V^\pm\Phi_i}^2 &= \left. \frac{\partial^2 \mathcal{L}_K}{\partial V^\pm \partial \Phi_i} \right|_{\Phi_i=0}, & M_{V^0\Phi_i}^2 &= \left. \frac{\partial^2 \mathcal{L}_K}{\partial V^0 \partial \Phi_i} \right|_{\Phi_i=0} \\ M_{W_3\Phi_i}^2 &= \left. \frac{\partial^2 \mathcal{L}_K}{\partial W_3 \partial \Phi_i} \right|_{\Phi_i=0}, & M_{W_8\Phi_i}^2 &= \left. \frac{\partial^2 \mathcal{L}_K}{\partial W_8 \partial \Phi_i} \right|_{\Phi_i=0}, & M_{B\Phi_i}^2 &= \left. \frac{\partial^2 \mathcal{L}_K}{\partial B \partial \Phi_i} \right|_{\Phi_i=0}, \end{aligned}$$

where Φ_i is each one of the scalar components in weak basis and the vectorial fields too. those terms give us the mass couplings with the bosonic sector. The bosons for the charged sector and the complex neutral bosons V^0 , results as a diagonal matrix so they already are in mass states so the masses for the physical Charged boson W^\pm , K^\pm and the neutral one V^0 are:

$$M_{W^\pm}^2 = \frac{g_L^2}{4} \nu_{ew}^2, \quad M_{K^\pm}^2 = \frac{g_L^2}{4} (u^2 + |w|^2), \quad M_{V^0, \bar{V}^0}^2 = \frac{g_L^2}{4} (u^2 + |\nu|^2). \quad (5.30)$$

Remembering that $\nu_{ew}^2 = |\nu|^2 + |w|^2$. As we can see in the previous expression we have that the new bosons K_μ and V_μ are sufficiently heavy in order to have consistence with low energy phenomenology, in contrast W^\pm acquire mass in the second transition at the electroweak energy scale.

On the other hand, the neutral gauge bosons exhibit mixing mass terms. This matrix is not singular, ensuring one massless boson (The Photon). Neutral sector have the following rotation to obtain the mass eigenstates:

$$\begin{pmatrix} A_\mu \\ Z_\mu^1 \\ Z_\mu^2 \end{pmatrix} = \mathbb{R}_{bos} \begin{pmatrix} W_\mu^3 \\ W_\mu^8 \\ B_\mu \end{pmatrix}, \quad (5.31)$$

where the matrix rotation is defined as:

$$\mathbb{R}_{bos} = \begin{pmatrix} S_W & \frac{1}{\sqrt{3}}C_W T_W & C_W \sqrt{A} \\ C_W C_\varphi & -\frac{1}{\sqrt{3}}S_W T_W C_\varphi + S_\varphi \sqrt{A} & -\frac{1}{\sqrt{3}}T_W S_\varphi - S_W C_\varphi \sqrt{A} \\ C_W S_\varphi & -\frac{1}{\sqrt{3}}S_W T_W S_\varphi - C_\varphi \sqrt{A} & \frac{1}{\sqrt{3}}T_W C_\varphi - S_W S_\varphi \sqrt{A} \end{pmatrix}, \quad (5.32)$$

where the angles satisfy the following relations:

$$S_W \equiv \sin W = \frac{g_X}{\sqrt{g_L^2 + \frac{4}{3}g_X^2}}, \quad S_\varphi \equiv \sin \varphi = \frac{\sqrt{3} g_L S_{2W}}{2 g_X C_W} \left(\frac{|\nu|^2 C_{2W}^2 - |w|^2}{|\nu|^2 C_{2W}^2 + |w|^2 + 4 C_W^4 u^2} \right),$$

$$\sqrt{A} = \sqrt{1 - \frac{1}{3}T_W^2},$$

The Z_μ^1 and Z_μ^2 are not the same as the Z and Z' boson, because there are a resulting mixing between them. Z_μ^1 can be identified as the phenomenological neutral weak boson and Z_μ^2 is a new heavy physical boson. As we can see, the angle φ that mix the components to obtain Z_μ^1 and Z_μ^2 is suppressed as $S_\varphi \propto \frac{\nu_{ew}}{u^2}$, thus, we can assume $\varphi = 0$ to obtain $Z_\mu^1 \rightarrow Z$ and $Z_\mu^2 \rightarrow Z'$.

Also, as in the SM, one has that $\frac{M_W^2}{M_Z^2} = C_W$, which allows to identify the angle $W \equiv \theta_W$ with the Weinberg angle, where $C_W^2 \approx 0.78$, so we have $\theta_W \approx 54^\circ$ and $T_W^2 \approx 11/6$. Then, we can identify Z as the neutral gauge boson of the standard model. The bosonic spectrum that acquire mass with the Goldstone bosons are presented in the table 5.2.

Table 5.2: Physical Vectorial Fields Spectrum

Physical Boson	Squared Mass
A_μ	0
W_μ^\pm	$\frac{g_L^2}{4} \nu_{ew}^2$
K_μ^\pm	$\frac{g_L^2}{4} (u^2 + w ^2)$
$K_\mu^0, \overline{K}_\mu^0$	$\frac{g_L^2}{4} (u^2 + \nu ^2)$
Z_μ^0	$\frac{g_L^2}{4C_W^2} \nu_{ew}^2$
$Z_\mu'^0$	$\frac{g_X^2}{3T_W^2} u^2$

By transforming both, the scalar and the vector boson basis into their physical mass eigenstates in the kinetic Lagrangian, we can obtain new CP violating coupling terms with phenomenological consequences, as discussed in chapter 6.

5.3 Physical Spectrum - Yukawa Sector

Due to the Z_2 symmetry which was chosen in the Higgs potential for the model, some terms in the Yukawa Lagrangian must be removed, spoiling the mass acquisition of the fermions. In particular, introducing this symmetry, all the light fermions (those terms coupled with ρ and η), become massless, so only the heavy fermions survive since they coupled with “ χ ”, that is invariant under the Z_2 transformation.

Thus, it is necessary to include additional Z_2 symmetries in the quarks sector in order to guarantee mass for all the fermionic fields. We choose the following symmetries:

Right-handed Sector :

$$U_R^{(1)} \rightarrow U_R^{(1)} \quad , \quad D_R^{(1)} \rightarrow -D_R^{(1)} \quad , \quad (5.33)$$

$$U_R^{(2,3)} \rightarrow -U_R^{(2,3)} \quad , \quad D_R^{(2,3)} \rightarrow D_R^{(2,3)} \quad , \quad (5.34)$$

Left-handed Sector :

$$Q_L^{(1)} \rightarrow Q_L^{(1)} \quad , \quad (5.35)$$

$$Q_L^{(2,3)} \rightarrow -Q_L^{(2,3)} \quad , \quad (5.36)$$

Taking into account the above symmetries we obtain in a explicitly way the following Yukawa Lagrangian:

$$\begin{aligned} \mathcal{L}_Y^Q = & \bar{Q}_L^{(1)} \left(\Gamma_{1,1}^{(D,\rho)} \rho D_R^{(1)} + \Gamma_{1,2}^{(J,\rho)} \rho J_R^{(2)} + \Gamma_{1,2}^{(U,\eta)} \eta U_R^{(2)} + \Gamma_{1,3}^{(U,\eta)} \eta U_R^{(3)} \right. \\ & \left. + \Gamma_{1,1}^{(J,\chi)} \chi J_R^{(1)} + \Gamma_{1,2}^{(D,\chi)} \chi D_R^{(2)} + \Gamma_{1,3}^{(D,\chi)} \chi D_R^{(3)} + \text{h.c.} \right) \\ & \bar{Q}_L^{(2)} \left(\Gamma_{2,1}^{(D,\chi)} \chi D_R^{(1)} + \Gamma_{2,2}^{(J,\chi)} \chi D_R^{(2)} + \Gamma_{2,1}^{(U,\eta)} \eta U_R^{(1)} + \Gamma_{2,3}^{(J,\eta)} \eta J_R^{(3)} \right. \\ & \left. + \Gamma_{2,1}^{(J,\rho)} \rho J_R^{(1)} + \Gamma_{2,2}^{(D,\rho)} \rho D_R^{(2)} + \Gamma_{2,3}^{(D,\rho)} \rho D_R^{(3)} + \text{h.c.} \right) \\ & \bar{Q}_L^{(3)} \left(\Gamma_{3,1}^{(U,\rho)} \rho^* U_R^{(1)} + \Gamma_{3,3}^{(J,\rho)} \rho^* J_R^{(3)} + \Gamma_{3,2}^{(U,\chi)} \chi^* U_R^{(2)} + \Gamma_{3,3}^{(U,\chi)} \chi^* U_R^{(3)} \right. \\ & \left. + \Gamma_{3,1}^{(J,\eta)} \eta^* J_R^{(1)} + \Gamma_{3,2}^{(D,\eta)} \eta^* D_R^{(2)} + \Gamma_{3,3}^{(D,\eta)} \eta^* D_R^{(3)} + \text{h.c.} \right) . \quad (5.37) \end{aligned}$$

Computing in the vacuum space we obtain the following mass matrix for the up sector and the down sector. that:

$$\mathbb{M}_U^2 = \left(\begin{array}{ccc|c} U_R^{(1)} & U_R^{(2)} & U_R^{(3)} & J_R^{(3)} \\ 0 & \Gamma_{1,2}^{(U,\eta)} w & \Gamma_{1,3}^{(U,\eta)} w & 0 \\ \Gamma_{2,1}^{(U,\eta)} w & 0 & 0 & \Gamma_{2,2}^{(J,\eta)} w \\ \Gamma_{3,1}^{(U,\rho)} \nu^* & 0 & 0 & \Gamma_{3,3}^{(J,\rho)} \nu^* \\ \hline 0 & \Gamma_{3,2}^{(U,\chi)} u & \Gamma_{3,3}^{(U,\chi)} u & 0 \end{array} \right) \begin{pmatrix} \bar{U}_L^{(1)} \\ \bar{U}_L^{(2)} \\ \bar{U}_L^{(3)} \\ \bar{J}_L^{(3)} \end{pmatrix}, \quad (5.38)$$

$$\mathbb{M}_D^2 = \left(\begin{array}{ccc|cc} D_R^{(1)} & D_R^{(2)} & D_R^{(3)} & J_R^{(1)} & J_R^{(1)} \\ \Gamma_{1,1}^{(D,\rho)} \nu & 0 & 0 & 0 & \Gamma_{1,2}^{(J,\rho)} \nu \\ 0 & \Gamma_{2,2}^{(D,\rho)} \nu & \Gamma_{2,3}^{(D,\rho)} \nu & \Gamma_{2,1}^{(J,\rho)} \nu & 0 \\ 0 & \Gamma_{3,2}^{(D,\eta)} w^* & \Gamma_{3,3}^{(D,\eta)} w^* & \Gamma_{3,1}^{(J,\eta)} w^* & 0 \\ \hline 0 & \Gamma_{1,2}^{(D,\chi)} u & \Gamma_{1,3}^{(D,\chi)} u & \Gamma_{1,1}^{(J,\chi)} u & 0 \\ \Gamma_{2,1}^{(D,\chi)} u & 0 & 0 & 0 & \Gamma_{2,2}^{(J,\chi)} u \end{array} \right) \begin{pmatrix} \bar{D}_L^{(1)} \\ \bar{D}_L^{(2)} \\ \bar{D}_L^{(3)} \\ \bar{J}_L^{(1)} \\ \bar{J}_L^{(2)} \end{pmatrix}. \quad (5.39)$$

In general those coupling parameters (Γ) are complex. Then we can obtain the physical states with a biunitary transformation.

After diagonalizing, the fields are obtained in mass eigenstates, however this sector exhibits many phenomenological aspects as the problem of Flavor Changing Neutral Currents (FCNC's) that must be controlled in this model, the hierarchy of quarks masses and the violation effects through the CKM matrix. These aspects are outside of the central purpose of this work but they can be considered for a later research.

Phenomenological Prospects and Conclusions

To explore the possibilities of detecting the CP violation effects in the Higgs sector, it is necessary to identify the contribution of the complex phase in the interactions of the scalar fields with the matter and radiation fields. These interactions are fundamentally described by three Lagrangians. The self interaction between the Higgs bosons are obtained directly from the Higgs potential, after rotation from weak basis into the mass eigenstates. The other source, is the kinetic sector of the Higgs Lagrangian which contains the couplings between the scalar fields with the vector fields. Finally, the couplings of fermions with scalars fields are obtained in the Yukawa Lagrangian.

Table 6.1: Possible Interaction Vertices

	Vertex	Standard Model	CPV 331 Model
I	$h - Z_\mu - Z_\mu$	$\frac{g_L M_Z}{C_W}$	$\frac{g_L M_Z}{C_W} C_{2\delta}$
II	$h - W_\mu^+ - W_\mu^-$	$g_L M_Z$	$g_L M_Z C_{2\delta}$
III	$h - h - Z_\mu - Z_\mu$	$\frac{g_L^2}{2 C_W}$	$\frac{g_L^2}{2 C_W}$
IV	$h - h - W_\mu^+ - W_\mu^-$	$\frac{g_L^2}{2}$	$\frac{g_L^2}{2}$
V	$h - h - h$	$\frac{3 g_L m_h^2}{2 M_W}$	$\frac{3 g_L m_h^2}{2 M_W} C'_{2\delta}$
VI	$h - h - h - h$	$\frac{3 g_L^2 m_h^2}{4 M_W^2}$	$\frac{3 g_L^2 m_h^2}{4 M_W^2}$

In order to indicate the different types of observables that could be used in order to verify the CP violation, let us consider, for example; trilinear and quartic couplings with the light Higgs boson “ h ”, which is associated with the physical boson “ h ” discovered in the LHC [25].

In particular, we can consider the self interactions for this boson, and its couplings with the weak charged bosons W^\pm and Z^0 . To compare our results, the table 6.1 shows these vertices as predicted by the Standard Model and for the CP not invariant 331 model.

As we can see in the table 6.1, not all the vertices give rise a phase factor related to CP violation effects. In particular the vertices that depends from the phase that breaks the CP symmetry are (I, II, V), all of them with the same factor “ $C_{2\delta}$ ”. As a consequence, the observables such as decay widths and effective cross sections in some processes of particle production and decays mediated by the Higgs boson, suffer a small suppression of the order of $C_{2\delta}$. For example, these vertices participate with important contributions in the production process of the Higgs boson, as shown in the figure 6.1 [26]:

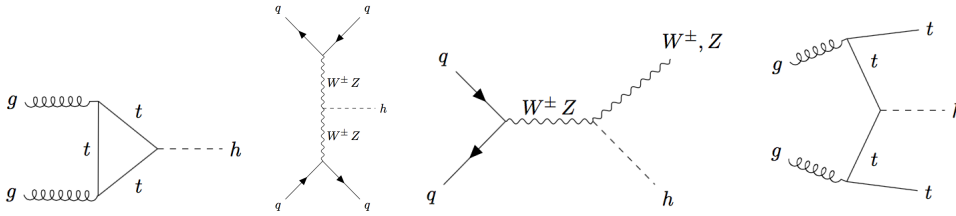


Figure 6.1

Although the main production channel for gluons fusion in the first diagram, does not contain any vertex from the table 6.1, the other diagrams gives us important information for the Higgs nature. If “ σ_{SM} ” is the effective cross section predicted by the standard model, diagrams which mediate vectorial bosons with the non invariant CP model are rescaled by a factor given by:

$$\sigma[VV \rightarrow h] = \sigma_{SM}[VV \rightarrow h]C_{2\delta}^2 \quad (6.1)$$

$$\sigma[V \rightarrow hV^*] = \sigma_{SM}[V \rightarrow hV^*]C_{2\delta}^2. \quad (6.2)$$

The Higgs boson decay can also give rise CP phase effects depending on the final states. For example, one of the dominant decay mode is: $h \rightarrow WW^*$, where the virtual weak boson mediate the decay to fermions, as shown in the figure 6.2. The width decay for the first vertex is also modified from the standard model with the factor:

$$\Gamma[h \rightarrow WW^*] = \Gamma_{SM}[h \rightarrow WW^*]C_{2\delta}^2. \quad (6.3)$$

At radiative corrections level, they can also show phase effects, that in principle could be measurable. For example, although the Higgs is neutral (it has not charge), it can exhibit photon decays through correction to one loop [27], as shown in the figure 6.3, that contains a vertex: $h - W - W$ which depends on the CP phase.

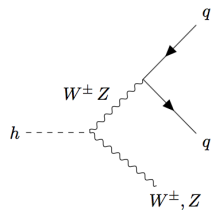


Figure 6.2

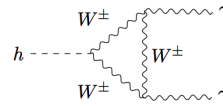


Figure 6.3

On the other hand, the quark sector introduces a complex phase originated from the complex nature of the Yukawa couplings, which is observable in the charged weak interactions through the CKM matrix [28]. These interactions produce CP violation processes in neutral mesonic systems, like the Kaon decays in the system $K_0 - \bar{K}_0$ [29]. In CP violation models in the Higgs sector, there is an additional contribution to the CKM phase, producing additional contributions to the observed process that violate CP symmetry and generating new process due to the couplings between Higgs bosons and fermions with an irreducible phase. For example, in collision processes at high energy systems as shown in the figure 6.1, the principal channel for Higgs production is through the Top Quarks loop. The CPV figure; the 331 model exhibits a contribution with the phase “ δ ” in the coupling: $t - t - h$, which has the following form:

$$\frac{m_t}{\sqrt{2}\nu} \bar{t}[(C_{2\delta} + iS_{2\delta}\gamma_5)h]t, \quad (6.4)$$

generating simultaneous contributions for both CP-odd and CP-even couplings, violating this discrete symmetry. It is evident that in the limit with $\delta \rightarrow 0$, the Higgs boson has a Yukawa pure CP-even coupling, reproducing the limit for a CP conservative model.

Finally, it is possible to evaluate the CP violation phase in an indirect way using low energy observables. One of them that impose strong restrictions, is the electric dipole moment for the electron. Contributions to the dipole moment of the electron in the models with CP phases in the Higgs sector occurs through 2-loop Barr-Zee corrections, as shown in the figure 6.4. Current experiments have not observed any electric dipole moment associated with the electron. The measurements impose strong constraints on the dipole moment at the order of [30]:

$$\left| \frac{d_e}{e} \right| < 8.7 \times 10^{-29} \text{ cm}. \quad (6.5)$$

At low energies, the two-loop diagrams lead an effective puntual interactio, whose value is expressed in terms of a dimensionless coefficient δ_e , it is the Wilson coefficient, such that the dipole moment is [31]:

$$|d_e| = \frac{2m_e}{\nu^2} |e\delta_e|. \quad (6.6)$$

The diagram includes CP violation contributions in various vertices: in the coupling $e - e - h$, also in the vertex: $h - t - t$ if it is with top quarks loop, $h - W^+ - W^-$ if it is with vector boson Loops, or: $h - H^+ - H^-$ if it is with the charged Higgs bosons. Finally, if the loop is with charged scalar bosons, a coupling depending on the phase between the Z bosons in the other side of the diagram is emerged. Each type of loop contributes to the value of the Wilson coefficient. Through the experimental constraints, we can obtain restrictions of the parameters of the model, including the CP violation phase.

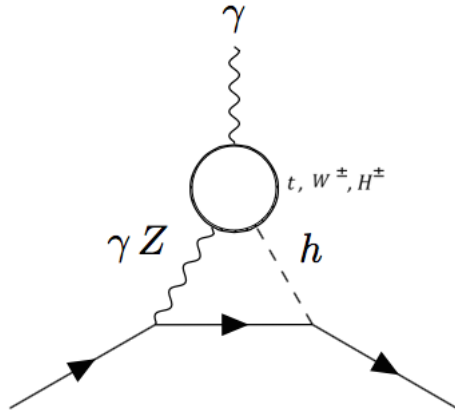


Figure 6.4

Notation

Here we are going to introduce some notation. We will adopt the unit system that is most convenient for high energy physics, where the speed of light provides the natural scale and quantum effects are not necessarily small, it is;

$$c = 1 = \hbar$$

We write four-vectors with upper and lower indices, where the metric tensor $g_{\mu\nu}$ is defined by;

$$g_{\mu\nu} = \text{diag}[1, -1, -1, -1]$$

which it can be used to raise or lower indices, as:

$$x^\mu = (t, \vec{x}) , \quad x_\mu = g_{\mu\nu}x^\nu = (t, -\vec{x})$$

with the usual convention of summing over repeated indices.

For the Dirac matrices we shall choose the representation;

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} , \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

where σ^i are the Pauli matrices. In addition we can define;

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Finally we have to defined the GellMann matrices as;

$$\begin{aligned}\lambda_1 &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ \lambda_4 &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \\ \lambda_6 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}\end{aligned}$$

Example CP invariance

Considering two complex scalar fields we can present here an example for a Higgs potential that preserve or not the CP symmetry.

CP invariant

Taken the following potential;

$$V_H = g_{abb}\phi_a\phi_b^2 + g_{abb}^*\phi_a^\dagger\phi_b^{\dagger 2} \quad (\text{B.1})$$

under the usual CP transformation, we have;

$$V_H \xrightarrow{CP} V'_H = g_{abb}\eta_a^*\eta_b^{*2}\phi_a^\dagger\phi_b^{\dagger 2} + g_{abb}^*\eta_a\eta_b^2\phi_a\phi_b^2 \quad (\text{B.2})$$

then, we can assume g as a complex parameter, and define a convenient phase in order to rotate ϕ basis.

$$g_{abb} = |g_{abb}|e^{i\alpha} \quad (\text{B.3})$$

$$\eta_a = e^{i2\alpha} \quad (\text{B.4})$$

$$\hat{\phi}_a = \phi_a e^{i\alpha} \quad (\text{B.5})$$

using the above definitions we can cancel the CP phases (η), thus we have a CP invariant potential with real parameters, as;

$$\tilde{V}_H = |g_{abb}| \left(\hat{\phi}_a\hat{\phi}_b^2 + \hat{\phi}_a^\dagger\hat{\phi}_b^{\dagger 2} \right) \quad (\text{B.6})$$

CP non-invariant

An example of a Higgs potential which is not invariant under CP in explicit way, could be;

$$\begin{aligned}
V_H &= g_{abb}\phi_a\phi_b^2 + g_{aab}\phi_a^2\phi_b + g_{aaa}\phi_a^3 + h.c. \\
V_H &= g_{abb}\phi_a\phi_b^2 + g_{abb}^*\phi_a^\dagger\phi_b^{\dagger 2} + g_{aab}\phi_a^2\phi_b + g_{aab}^*\phi_a^{\dagger 2}\phi_b^\dagger + g_{aaa}\phi_a^3 + g_{aaa}^*\phi_a^{\dagger 3} \\
V_H &= A\phi_a\phi_b^2 + A^*\phi_a^\dagger\phi_b^{\dagger 2} + B\phi_a^2\phi_b + B^*\phi_a^{\dagger 2}\phi_b^\dagger + C\phi_a^3 + C^*\phi_a^{\dagger 3}
\end{aligned} \tag{B.7}$$

applying CP transformation, we have;

$$CPV_H P^\dagger C^\dagger = A\eta_a^*\eta_b^{*2}\phi_a^\dagger\phi_b^{\dagger 2} + A^*\eta_a\eta_b^2\phi_a\phi_b^2 \tag{B.8}$$

$$+ B\eta_a^{*2}\eta_b^*\phi_a^{\dagger 2}\phi_b^\dagger + B^*\eta_a^2\eta_b\phi_a^2\phi_b \tag{B.9}$$

$$+ C\eta_a^{*3}\phi_a^{\dagger 3} + C^*\eta_a^3\phi_a^3 \tag{B.10}$$

and assuming complex parameters and defining CP phases, as;

$$\begin{aligned}
A &= |A|e^{ia} \quad , \quad B = |B|e^{ib} \quad , \quad C = |C|e^{ic} \\
\eta_a &= e^{i\alpha} \quad , \quad \eta_b = e^{i\beta}
\end{aligned}$$

solving equivalence equations between (A.7 and A.8,9,10), and taking the first two terms we have;

$$\begin{aligned}
|A|e^{ia} &= |A|e^{-ia}e^{i\alpha}e^{2i\beta} & |B|e^{ib} &= |B|e^{-ib}e^{2i\alpha}e^{i\beta} \\
a &= \alpha + 2\beta - a & b &= 2\alpha + \beta - b \\
2a &= \alpha + 2\beta & 2b &= 2\alpha + \beta
\end{aligned}$$

we find a possible solution with CP invariance

$$\begin{cases} \alpha = \frac{2}{3}(2b - a) \\ \beta = \frac{2}{3}(2a - b) \end{cases}$$

Although, if we include the third term

$$\begin{aligned}
|C|e^{ic} &= |C|e^{-ic}e^{3i\alpha} \\
c &= 3\alpha - c \\
2c &= 3\alpha
\end{aligned}$$

There is a particular solution that implies a relation between the parameter phases, so it is not CP invariant, due to the invariance must be independent from parameter phases relations.

$$\begin{cases} \alpha = \frac{2}{3}c \\ \alpha = \frac{2}{3}(2b - a) \\ \beta = \frac{2}{3}(2a - b) \end{cases} \rightarrow \boxed{c = 2b - a}$$

Structure constants for SU(3) group

The elements for the structure constants which are not zero, are given by:

$$\begin{aligned}
 f^{123} &= 1 \\
 f^{147} &= -f^{156} = f^{246} = f^{257} = f^{345} = -f^{367} = \frac{1}{2} \\
 f^{458} &= f^{678} = \frac{\sqrt{3}}{2}
 \end{aligned}$$

d-constants (symmetric) are:

$$\begin{aligned}
 d^{118} &= d^{228} = d^{338} = -d^{888} = \frac{1}{\sqrt{3}} \\
 d^{448} &= d^{558} = d^{668} = d^{778} = -\frac{1}{2\sqrt{3}} \\
 d^{146} &= d^{157} = -d^{247} = d^{256} = d^{344} = d^{355} = -d^{366} = -d^{377} = \frac{1}{2}
 \end{aligned}$$

Commutation Relations for Vacuum State

According to the scheme for the spontaneous symmetry breaking we must satisfy the following commutation relations for the vacuum state;

First Transition ;

$$1 \text{ SSB} : \begin{cases} [G_{1,2,3}, \langle \Phi_1 \rangle] = 0, & \text{(D.1a)} \\ [\beta G_8 + X, \langle \Phi_1 \rangle] = 0, & \text{(D.1b)} \\ [G_{4,5,6,7}, \langle \Phi_1 \rangle] \neq 0, & \text{(D.1c)} \\ [\beta G_8 - X, \langle \Phi_1 \rangle] \neq 0 & \text{(D.1d)} \end{cases}$$

Second Transition ;

$$2 \text{ SSB} : \begin{cases} [G_1, \langle \Phi_2 \rangle] \neq 0, & \text{(D.2a)} \\ [G_2, \langle \Phi_2 \rangle] \neq 0, & \text{(D.2b)} \\ [G_3 - \beta G_8 - X, \langle \Phi_2 \rangle] \neq 0, & \text{(D.2c)} \\ [G_3 + \beta G_8 + X, \langle \Phi_2 \rangle] = 0 & \text{(D.2d)} \end{cases}$$

Choosing $\beta = 1/\sqrt{3}$ which is our case, and considering the first transition with the scalar field χ in general we assume vacuum value for all the components;

$$\langle \chi \rangle = \begin{pmatrix} \nu_{\chi_1} \\ \nu_{\chi_2} \\ \nu_{\chi_3} \end{pmatrix} \tag{D.3}$$

applying the first condition (eq. D.1a), we have;

$$\nu_{\chi_1} = 0, \nu_{\chi_2} = 0, \nu_{\chi_3} : \text{Free Par.} \quad (\text{D.4})$$

In the other hand for condition (eq. D.1b) we obtain;

$$\left(-\frac{\beta}{\sqrt{3}} + X_\chi\right) \nu_{\chi_3} = 0 \rightarrow \text{if } \nu_{\chi_3} \neq 0 \text{ then } \frac{1}{3} = X_\chi$$

The last relation give us X_χ quantum number for χ , and the broken generators (eq. D.1d and D.1c) ensure us that $\nu_{\chi_3} \neq 0$ for this choice, so we have the vacuum value for χ

$$\langle \chi \rangle = \begin{pmatrix} 0 \\ 0 \\ \nu_{\chi_3} \end{pmatrix} \quad (\text{D.5})$$

In a similar way for the second transition we take for example;

$$\langle \rho \rangle = \begin{pmatrix} \nu_{\rho_1} \\ \nu_{\rho_2} \\ \nu_{\rho_3} \end{pmatrix} \quad (\text{D.6})$$

Using the condition (eq. D.2d) we obtain the followig relations for general β at left and $\beta = 1/\sqrt{3}$ taking also $X_\rho = 1/3$ at the right side¹;

$$\begin{aligned} I & \quad \left(\frac{1}{2} + \frac{\beta}{2\sqrt{3}} + X\right) \nu_{\rho_1} = 0 \quad \xrightarrow[\text{X}_\rho=1/3]{\beta=1/\sqrt{3}} \quad \nu_{\rho_1} = 0 \\ II & \quad \left(-\frac{1}{2} + \frac{\beta}{2\sqrt{3}} + X\right) \nu_{\rho_2} = 0 \quad \longrightarrow \quad \nu_{\rho_2} : \text{Free Par.} \\ III & \quad \left(-\frac{\beta}{\sqrt{3}} + X\right) \nu_{\rho_3} = 0 \quad \longrightarrow \quad \nu_{\rho_3} : \text{Free Par.} \end{aligned}$$

By conditions (eq. D.2a and D.2b) we have that; at least $\nu_{\rho_1} \neq 0$ or $\nu_{\rho_2} \neq 0$, it is

$$IV \quad \begin{pmatrix} \nu_{\rho_1} \\ \nu_{\rho_2} \\ 0 \end{pmatrix} \neq 0 \quad \longrightarrow \quad \nu_{\rho_2} \neq 0$$

¹This value for X can be obtained considering a general β and evaluating the new relations (I, II, III, IV, V, VI, VII).

Finally applying the condition (eq. D.2c) we have the following constrains, where the conditions must be satisfied as V or VI or VII;

$$\begin{aligned}
 V & \quad \left(\frac{1}{2} - \frac{\beta}{2\sqrt{3}} - X \right) \nu_{\rho_1} \neq 0 \quad \longrightarrow \quad \nu_{\rho_2} \neq 0 \text{ or } \nu_{\rho_3} \neq 0 \\
 VI & \quad \left(-\frac{1}{2} - \frac{\beta}{2\sqrt{3}} - X \right) \nu_{\rho_2} \neq 0 \quad \longrightarrow \quad -\nu_{\rho_2} \neq 0 \\
 VII & \quad \left(\frac{\beta}{\sqrt{3}} - X \right) \nu_{\rho_3} \neq 0 \quad \longrightarrow \quad \nu_{\rho_1} \neq 0 \text{ or } \nu_{\rho_2} \neq 0
 \end{aligned}$$

so the choice that satisfy the constrains are; $\nu_{\rho_1} = 0$, by (I), ν_{ρ_3} as a free parameter, by (III), and finally $\nu_{\rho_2} \neq 0$, by the common condition in (IV, V, VI and VII).

$$\langle \rho \rangle = \begin{pmatrix} 0 \\ \nu_{\rho_2} \neq 0 \\ \nu_{\rho_3} \end{pmatrix} \quad (D.7)$$

In particular we can take $\nu_{\rho_3} = 0$, as we need it after SU(3) rotation.

For the case in which we choose $\nu_{\rho_1} \neq 0$ with $\beta = 1/\sqrt{3}$ for conditions (I - VII) is the scalar field corresponding with η , where;

$$\langle \eta \rangle = \begin{pmatrix} \nu_{\eta_1} \\ 0 \\ 0 \end{pmatrix}, \quad X_\eta = -\frac{2}{3} \quad (D.8)$$

Matrix Transformation for SU(3)

Using the BCH relation for special unitary groups SU(N) [21](spectral theorem).

$$f(\mathbb{M}) = \sum_{n=0}^{N-1} f_n(\mathbb{M})\mathbb{M}^n \quad (\text{E.1})$$

por ejemplo para la exponencial ($f(\mathbb{M}) := e^{i\mathbb{M}}$)

$$e^{i\mathbb{M}} = \sum_{m=0}^{\infty} \frac{(-i\mathbb{M})^m}{m!} = \sum_{n=0}^{N-1} f_n(\mathbb{M})\mathbb{M}^n \quad (\text{E.2})$$

where; \mathbb{M} is an hermitian matrix, such that;

$$f(\mathbb{M}) = \sum_{k=1}^N f(m_k)\mathbb{P}_k \quad (\text{E.3})$$

with (m_k) as the eigenvalues and $(\mathbb{P}_k = |m_k\rangle\langle m_k|)$ are the projection operators which can set an eigenspace using the eigenvectors. Matrices \mathbb{P}_k could be set as a power series of (\mathbb{M}) , from the following expression;

$$\mathbb{P}_k = \prod_{j \neq k} \frac{\mathbb{M} - m_j \mathbb{I}}{m_k - m_j} = \sum_{n=0}^{N-1} P_{kn} \mathbb{M}^n \quad (\text{E.4})$$

Thus, the coefficient from equation E.2, can be writren as;

$$f_n(\mathbb{M}) = \sum_{k=1}^N P_{kn} f(m_k) \quad (\text{E.5})$$

Due to; our hermitian matrix is the product between the rotation angle α with the group generators; $\mathbb{M} = \vec{\alpha}_a \cdot \vec{T}$. We can redefine the matrix in order to become it as unitary ann take as factor the absolute value for α , it is $\mathbb{M}' = |\alpha|(\hat{n} \cdot \vec{T}) = \alpha \mathbb{M}$. Thus, the powers for the matrix \mathbb{M} are:

$$\mathbb{M}^0 = \mathbb{I} \quad (\text{E.6})$$

$$(\hat{n} \cdot \vec{T}) = \mathbb{M}^1 = \frac{1}{2} \begin{pmatrix} n_3 + \frac{1}{\sqrt{3}}n_8 & n_1 - in_2 & n_4 - in_5 \\ n_1 + in_2 & -n_3 + \frac{1}{\sqrt{3}}n_8 & n_6 - in_7 \\ n_4 + in_5 & n_6 + in_7 & -\frac{2}{\sqrt{3}}n_8 \end{pmatrix} \quad (\text{E.7})$$

$$\mathbb{M}^2 = \frac{1}{4} \begin{pmatrix} \frac{1}{3}n_8^2 + A + B + C & U_{12} & U_{13} \\ U_{12}^* & \frac{1}{3}n_8^2 + A - B + D & U_{23} \\ U_{13}^* & U_{23}^* & C + D + \frac{4}{3}n_8^2 \end{pmatrix} \quad (\text{E.8})$$

where ;

$$A = n_1^2 + n_2^2 + n_3^2, \quad B = \frac{2}{\sqrt{3}}n_3n_8, \quad C = n_4^2 + n_5^2, \quad D = n_6^2 + n_7^2$$

$$U_{12} = \frac{2}{\sqrt{3}}n_8(n_1 - in_2) + n_4n_6 + n_5n_7 - i(n_5n_6 - n_4n_7)$$

$$U_{13} = n_3(n_4 - in_5) - \frac{1}{\sqrt{3}}n_8(n_4 - in_5) + n_1n_6 - n_2n_7 - i(n_2n_6 + n_1n_7)$$

$$U_{23} = -n_3(n_6 - in_7) - \frac{1}{\sqrt{3}}n_8(n_6 - in_7) + n_4n_1 + n_5n_2 - i(n_5n_1 - n_4n_2)$$

Changing the parameter space from 8 real parameters $(n_{1,2,\dots,8})$ to 3 complex parameters (b, c, e) and 2 real parameters (a, d) .

$$\mathbb{M} = \frac{1}{2} \begin{pmatrix} n_3 + \frac{1}{\sqrt{3}}n_8 & n_1 - in_2 & n_4 - in_5 \\ n_1 + in_2 & -n_3 + \frac{1}{\sqrt{3}}n_8 & n_6 - in_7 \\ n_4 + in_5 & n_6 + in_7 & -\frac{2}{\sqrt{3}}n_8 \end{pmatrix} = \begin{pmatrix} a & b & c \\ b^* & d & e \\ c^* & e^* & -(a+d) \end{pmatrix} \quad (\text{E.9})$$

$$\mathbb{M}^2 = \begin{pmatrix} a^2 + b^2 + c^2 & b(a+d) + ce^* & -cd + be \\ b^*(a+d) + c^*e & d^2 + b^2 + e^2 & -ed + cb^* \\ -c^*d + b^*e^* & -e^*d + c^*b & c^2 + e^2 + (a+d)^2 \end{pmatrix} \quad (\text{E.10})$$

Given that matrix \mathbb{M} satisfy the Cayley-Hamilton theorem ($M^3 = I\det(M) + \frac{1}{2}M\text{tr}(M^2)$). Applying the spectral descomposition for our group SU(3), that is considering ($N = 3$), we can apply the general form to find the coefficients f_n , which can written as [?]:

$$e^{i\alpha\mathbb{M}} = \sum_{k=0,1,2} \mathcal{F} \left[\mathbb{M}^2 + \frac{2}{\sqrt{3}} \sin(\phi + 2\pi k/3) \mathbb{M} - \frac{1}{3} (1 + 2 \cos(2\phi + 4\pi k/3)) \mathbb{I} \right] \quad (\text{E.11})$$

where;

$$\mathcal{F} = \frac{e^{\frac{2}{\sqrt{3}}i\alpha \sin(\phi+2\pi k/3)}}{1 - 2 \cos(2\phi + 4\pi k/3)}$$

$$\phi = \frac{1}{3} \left[\arccos \left(\frac{3\sqrt{3}}{2} \det(\mathbb{M}) \right) - \frac{\pi}{2} \right]$$

Applying the summation and matching up identity, linear and quadratic terms, is possible to obtain coefficients (f_n), which in general can be complex factors.

$$e^{i\alpha\mathbb{M}} = f_0 \mathbb{I} + f_1 \mathbb{M} + f_2 \mathbb{M}^2$$

Finally, after the matrices addition, using equations E.9 and E.10; we can get the transformation in matricial form ($e^{i\alpha\mathbb{M}} = \mathbb{T}$). This matrix can be written as:

$$\mathbb{T} = \begin{pmatrix} f_0 + f_1 a + f_2 (a^2 + b^2 + c^2) & f_1 b + f_2 (b(a+d) + ce^*) & f_1 c + f_2 (-cd + be) \\ f_1 b^* + f_2 (b^*(a+d) + c^*e) & f_0 + f_1 d + f_2 (d^2 + b^2 + e^2) & f_1 e + f_2 (-ed + cb^*) \\ f_1 c^* + f_2 (-c^*d + b^*e^*) & f_1 e^* + f_2 (-e^*d + c^*b) & f_0 - f_1 (a+d) + f_2 (c^2 + e^2 + (a+d)^2) \end{pmatrix} \quad (\text{E.12})$$

Making a new parametric representation, the transformation matrix is:

$$\mathbb{T} = \frac{1}{2} \begin{pmatrix} 2T_1 & m_1 + n_1 & m_2 + n_2 \\ m_1 - n_1 & 2T_2 & m_3 + n_3 \\ m_2 - n_2 & m_3 - n_3 & 2T_3 \end{pmatrix} \quad (\text{E.13})$$

where;

$$\begin{aligned} m_1 &= 2[f_1 + f_2(a+d)]\text{Re}(b) + 2f_2\text{Re}(ce^*) \\ n_1 &= 2[f_1 + f_2(a+d)]i\text{Im}(b) + 2f_2i\text{Im}(ce^*) \\ m_2 &= 2[f_1 - f_2d]\text{Re}(c) + 2f_2\text{Re}(be^*) \\ n_2 &= 2[f_1 - f_2d]i\text{Im}(c) + 2f_2i\text{Im}(be^*) \\ m_3 &= 2[f_1 - f_2d]\text{Re}(e) + 2f_2\text{Re}(cb^*) \\ n_3 &= 2[f_1 - f_2d]i\text{Im}(e) + 2f_2i\text{Im}(cb^*) \\ T_1 &= \mathbb{T}_{11}, \quad T_2 = \mathbb{T}_{22}, \quad T_3 = \mathbb{T}_{33} \end{aligned}$$

SU(3) Rotation over the Scalar Fields

If we apply \mathbb{T} over scalar field (η), it must remain invariant the second and third component.

$$\begin{aligned} \eta' &= \mathbb{T}\eta \\ \begin{pmatrix} \omega' \\ 0 \\ 0 \end{pmatrix} &= \frac{1}{2} \begin{pmatrix} 2T_1 & m_1 + n_1 & m_2 + n_2 \\ m_1 - n_1 & 2T_2 & m_3 + n_3 \\ m_2 - n_2 & m_3 - n_3 & 2T_3 \end{pmatrix} \begin{pmatrix} \omega \\ 0 \\ 0 \end{pmatrix} \end{aligned} \quad (\text{F.1})$$

Considering the mentioned invariance, we obtain the following relations;

$$\omega' = 2T_1\omega, \quad m_1 = n_1, \quad m_2 = n_2 \quad (\text{F.2})$$

Taking above equations and adjusting the transformation matrix, we have;

$$\mathbb{T}' = \frac{1}{2} \begin{pmatrix} 2T_1 & 2m_1 & 2m_2 \\ 0 & 2T_2 & m_3 + n_3 \\ 0 & m_3 - n_3 & 2T_3 \end{pmatrix} \quad (\text{F.3})$$

As to the scalar field (ρ), we want to rotate the triplet to obtain the VEV just in the second component¹.

$$\begin{aligned} \rho' &= \mathbb{T}'\rho \\ \begin{pmatrix} 0 \\ \nu' \\ 0 \end{pmatrix} &= \frac{1}{2} \begin{pmatrix} 2T_1 & 2m_1 & 2m_2 \\ 0 & 2T_2 & m_3 + n_3 \\ 0 & m_3 - n_3 & 2T_3 \end{pmatrix} \begin{pmatrix} 0 \\ \nu_1 \\ \nu_2 \end{pmatrix} \end{aligned} \quad (\text{F.4})$$

¹If we left the VEV in the third component, as consequence we have massless terms in the Yukawa sector and we must to ensure that all fermions acquire mass.

Again we can obtain the following relations;

$$\frac{\nu_1}{\nu_2} = -\frac{m_2}{m_1}, \quad T_2 = \frac{\nu'}{\nu_1} + \frac{(m_3 + n_3)m_1}{2m_2}, \quad T_3 = \frac{(m_3 - n_3)m_2}{2m_1} \quad (\text{F.5})$$

Adjusting the transformation matrix;

$$\mathbb{T}'' = \frac{1}{2} \begin{pmatrix} 2T_1 & 2m_1 & 2m_2 \\ 0 & \frac{2\nu'}{\nu_1} + \frac{(m_3+n_3)m_1}{m_2} & m_3 + n_3 \\ 0 & m_3 - n_3 & \frac{(m_3-n_3)m_2}{m_1} \end{pmatrix} \quad (\text{F.6})$$

Due to scalar field χ transforms as; $e^{-i2\alpha\rho}$, then we have a new transformation matrix that satisfy; $\mathbb{T}''' = \mathbb{T}''^{-2}$, thus the determinant must be zero for \mathbb{T}'' , it is ($\det \mathbb{T}'' \neq 0$).

$$\det \mathbb{T}'' = \frac{T_1 m_2 \nu'}{2m_1 \nu_1} (m_3 - n_3) \rightarrow m_3 \neq n_3 \quad (\text{F.7})$$

Analogously this new matrix must satisfy;

$$\begin{aligned} \chi' &= \mathbb{T}''' \chi \\ \begin{pmatrix} 0 \\ 0 \\ u' \end{pmatrix} &= \mathbb{T}''' \begin{pmatrix} 0 \\ 0 \\ u \end{pmatrix} \rightarrow 0 = \mathbb{T}'''_{23} u \end{aligned} \quad (\text{F.8})$$

Above relations generate the following conditions;

$$\begin{aligned} 0 &= \mathbb{T}'''_{23} u \\ 0 &= -(m_3 + n_3) \frac{m_1^2 \nu_1^2}{m_2^2 \nu'^2} \left[(m_3 + n_3) \frac{m_1}{m_2} + \frac{2\nu'}{\nu_1} + (m_3 - n_3) \frac{m_2}{m_1} \right] / (m_3 - n_3)^2 \\ 0 &= (m_3 + n_3) \left[(m_3 + n_3) \frac{m_1}{m_2} + \frac{2\nu'}{\nu_1} + (m_3 - n_3) \frac{m_2}{m_1} \right] \end{aligned} \quad (\text{F.9})$$

For convenience we can choose ($m_3 = -n_3$) as a particular solution in which the transformed field ($\mathbb{T}''' \chi$), give us the following results;

$$u' = \left(\frac{m_1}{m_2 m_3} \right)^2 u, \quad T_1 = -\frac{m_2 m_3}{m_1} \quad (\text{F.10})$$

Finally using above relations and equation F.3, the final matrix transformation for SU(3) gauge symmetry, rotates the scalar field (ρ) in order to leave only VEV in the second component and remain invariant the position for the other VEV's.

$$\mathbb{T} = \begin{pmatrix} \frac{m_3 \nu_1}{\nu_2} & m_1 & -\frac{m_1 \nu_1}{\nu_2} \\ 0 & \frac{\nu'}{\nu_1} & 0 \\ 0 & m_3 & -\frac{m_3 \nu_1}{\nu_2} \end{pmatrix}, \quad \nu' = \left(\frac{\nu_2}{m_3} \right)^2 \frac{1}{\nu_1} \quad (\text{F.11})$$

Mass States Diagonalization for Charged Sector

G.1 Mass Matrix for Charged Sector

Applying the second derivative over the higgs potential, we can obtain the following mass matrix for the charged sector;

$$M_C^2 = \begin{pmatrix} M_{C1}^2 & 0 \\ 0 & M_{C2}^2 \end{pmatrix} \quad (\text{G.1})$$

where;

$$M_{C1}^2 = \begin{pmatrix} \frac{1}{2} l_7 w_1^2 + \frac{l_7 v_2^2 w_1^2}{2 v_1^2} + \frac{\sqrt{2} f v_1 w_1}{2 u} + \frac{\sqrt{2} f v_2^2 w_1}{2 u v_1} & \frac{1}{2} \left(\frac{l_7 u w_1}{v_1} + \sqrt{2} f \right) (v_1 - i v_2) \\ \frac{1}{2} \left(\frac{l_7 u w_1}{v_1} + \sqrt{2} f \right) (v_1 + i v_2) & \frac{1}{2} l_7 u^2 + \frac{\sqrt{2} f u v_1}{2 w_1} \end{pmatrix}$$

$$M_{C2}^2 = \begin{pmatrix} \frac{1}{2} l_9 w_1^2 + \frac{l_9 v_2^2 w_1^2}{2 v_1^2} + \frac{\sqrt{2} f u w_1}{2 v_1} & \frac{1}{2} l_9 v_1 w_1 + \frac{l_9 v_2^2 w_1}{2 v_1} + \frac{1}{2} \sqrt{2} f u \\ \frac{1}{2} l_9 v_1 w_1 + \frac{l_9 v_2^2 w_1}{2 v_1} + \frac{1}{2} \sqrt{2} f u & \frac{1}{2} l_9 v_1^2 + \frac{1}{2} l_9 v_2^2 + \frac{\sqrt{2} f u w_1}{2 w_1} \end{pmatrix}$$

Both matrices are singular, so we can ensure at least two Goldstone boson for each submatrix. The Eigenvalues could be found through the polynomial charecteristical equation. So we can obtain the following Eigenvalues

$$M_{C1}^2 : \begin{cases} \lambda_1 = 0 \\ \lambda_3 = \frac{1}{2} \left(l_7 + \sqrt{2} f \frac{v_1}{u w_1} \right) \left(u^2 + \frac{w_1^2}{v_1^2} (v_1^2 + v_2^2) \right) \end{cases} \quad (\text{G.2a})$$

$$(\text{G.2b})$$

$$M_{C2}^2 : \begin{cases} \lambda_2 = 0 \\ \lambda_4 = \frac{1}{2} \left(l_9(v_1^2 + v_2^2) + \sqrt{2}f \frac{uv_1}{w_1} \right) \left(1 + \frac{w_1^2}{v_1^2} \right) \end{cases} \quad (\text{G.3a})$$

G.2 Eigenvectors and Rotation Matrix

Using the characteristic equation for eigenvectors, as a matrix multiplication on the left hand side ($M_{C_i}^2 \vec{V}_j = \lambda_j \vec{V}_j$); we can find the following results

$$\vec{V}_1 = \left(1, -\frac{(v_1 + i v_2)w_1}{uv_1} \right)$$

$$\vec{V}_3 = \left(1, \frac{uv_1^2 + i uv_1 v_2}{(v_1^2 + v_2^2)w_1} \right)$$

Those vectors must be orthonormal between them; so applying this condition and taking the inverse matrix rotation with the eigenvectors as rows, we have;

$$\mathbb{R}_1 = \begin{pmatrix} \frac{\sqrt{u^2 - |w|^2}}{u} & \frac{|w|}{|\nu|} \left(\frac{v_1 - i v_2}{u} \right) \\ -\frac{|w|}{|\nu|} \left(\frac{v_1 + i v_2}{u} \right) & \frac{\sqrt{u^2 - |w|^2}}{u} \end{pmatrix}$$

This matrix rotation could be parametrized with the angles (α and δ), where;

$$\sin \alpha = -\frac{|w|}{u}; \quad |w| = \frac{w_1}{v_1} |\nu| \quad (\text{G.4})$$

$$e^{i\delta} = \frac{v_1 + i v_2}{|\nu|}; \quad |\nu| = \sqrt{v_1^2 + v_2^2} \quad (\text{G.5})$$

Following the same procedure for the other submatrix, we obtain the following eigenvectors;

$$\vec{V}_2 = \left(1, -\frac{w_1}{v_1} \right)$$

$$\vec{V}_4 = \left(1, \frac{v_1}{w_1} \right)$$

with matrix rotation;

$$\mathbb{R}_2 = \frac{1}{\nu_{ew}} \begin{pmatrix} |\nu| & -|w| \\ |w| & |\nu| \end{pmatrix}$$

and the θ angle can be defined as;

$$\sin \theta = -\frac{|w|}{\nu_{ew}}; \quad \nu_{ew} = \sqrt{|w|^2 + |v|^2} \quad (\text{G.6})$$

Mass States Diagonalization for Neutral Sector

As was explained in section 5.1.3, the neutral sector can be taken as two submatrices in a convenient basis; $(\xi_{\chi_2}, \zeta_{\chi_2}, \xi_{\rho_3}, \zeta_{\rho_3}, \zeta_{\chi_3}, \zeta_{\rho_2}, \zeta_{\eta}, \xi_{\rho_2}, \xi_{\eta}, \xi_{\chi_3})$.

H.1 Mass Matrix - \mathbb{M}_a

The mass matrix \mathbb{M}_a is obtained in the basis; $(\xi_{\chi_2}, \zeta_{\chi_2}, \xi_{\rho_3}, \zeta_{\rho_3})$. This matrix is singular so again there is at least one Goldstone boson, and we can identify a submatrices with a particular hierarchy energy scales so we can express that as;

$$\mathbb{M}_a = \begin{pmatrix} A & B \\ B^T & C \end{pmatrix}$$

where;

$$A = \begin{pmatrix} l_{10}(v_1^2 - v_2^2) + |v|^2 \left(\frac{1}{2} l_8 + \frac{\sqrt{2}fw_1}{2uv_1} \right) & 2l_{10}v_1v_2 \\ 2l_{10}v_1v_2 & -l_{10}(v_1^2 - v_2^2) + |v|^2 \left(\frac{1}{2} l_8 + \frac{\sqrt{2}fw_1}{2uv_1} \right) \end{pmatrix}$$

$$B = \begin{pmatrix} l_{10}uv_1 + \frac{1}{2} l_8uv_1 + \frac{1}{2} \sqrt{2}fw_1 & -l_{10}uv_2 + \frac{1}{2} l_8uv_2 + \frac{\sqrt{2}fv_2w_1}{2v_1} \\ l_{10}uv_2 + \frac{1}{2} l_8uv_2 + \frac{\sqrt{2}fv_2w_1}{2v_1} & l_{10}uv_1 - \frac{1}{2} l_8uv_1 - \frac{1}{2} \sqrt{2}fw_1 \end{pmatrix}$$

$$C = \begin{pmatrix} l_{10}u^2 + \frac{1}{2} l_8u^2 + \frac{\sqrt{2}fuw_1}{2v_1} & 0 \\ 0 & -l_{10}u^2 + \frac{1}{2} l_8u^2 + \frac{\sqrt{2}fuw_1}{2v_1} \end{pmatrix}$$

As we can see matrix C is proportional to energy scale for the first transition ($C \propto u^2$); in the other hand the B matrix is a medium scale ($B \propto u$) and finally we have that A matrix is at electroweak scale ($A \propto \nu_{ew}$), so we have the following hierarchy;

$$C \gg B \gg A$$

Then we have can do a block diagonalization process, considering an unitary matrix rotation [?];

$$V = \begin{pmatrix} 1 & F \\ -F^T & 1 \end{pmatrix}$$

where 1 is the identity matrix and F is a submatrix that must satisfy ($F \ll 1$), so keeping only up to linear terms on F , the matrix rotation has the form;

$$V^T \mathbb{M}_a V = \begin{pmatrix} A - BF^T - FB^T & B + AF - FC \\ B^T + F^T A - CF^T & C + B^T F + F^T B \end{pmatrix} = \begin{pmatrix} \mathbb{D}_{a1} & 0 \\ 0 & \mathbb{D}_{a2} \end{pmatrix}$$

But considering the hierarchy conditions we can obtain that;

$$F \approx BC^{-1}$$

Then, we have the following approximations;

$$\mathbb{D}_{a1} \approx A - BC^{-1}B^T \quad (\text{H.1})$$

$$\mathbb{D}_{a2} \approx C \quad (\text{H.2})$$

finally , those matrices can be diagonalized independently, so we have

$$\mathbb{D}_{a1} : \begin{cases} \lambda_5 = 0 \\ \lambda_6 = 0 \end{cases} \quad (\text{H.3a})$$

$$(\text{H.3b})$$

$$\mathbb{D}_{a2} : \begin{cases} \lambda_7 = \frac{u^2}{2}(l_8 + 2l_{10}) + \frac{f u w_1}{\sqrt{2} v_1} \\ \lambda_8 = \frac{u^2}{2}(l_8 - 2l_{10}) + \frac{f u w_1}{\sqrt{2} v_1} \end{cases} \quad (\text{H.4a})$$

$$(\text{H.4b})$$

H.2 Mass Matrix - \mathbb{M}_b

For the second submatrix in the neutral sector in tthe basis $(\zeta_{\chi_3}, \zeta_{\rho_2}, \zeta_{\eta}, \xi_{\rho_2}, \xi_{\eta}, \xi_{\chi_3})$, we have the following structure for the matrix mass;

$$\mathbb{M}_b = \begin{pmatrix} M_{b_{11}} & D & 0 \\ D^T & \mathbb{H} & E \\ 0 & E^T & M_{b_{66}} \end{pmatrix} \quad (\text{H.5})$$

where;

$$\begin{aligned} M_{b_{11}} &= \frac{\sqrt{2}fw_1}{2uv_1}|v|^2 \\ M_{b_{66}} &= 2l_1u^2 + \frac{\sqrt{2}fw_1}{2uv_1}|v|^2 \\ D &= \left(\frac{1}{2}\sqrt{2}fw_1, \frac{1}{2}\sqrt{2}fv_1, -\frac{\sqrt{2}fv_2w_1}{2v_1}, \frac{1}{2}\sqrt{2}fv_2 \right) \\ E^T &= \left(l_4uv_2 - \frac{\sqrt{2}fv_2w_1}{2v_1}, -\frac{l_5uv_2w_1}{v_1} + \frac{\sqrt{2}fv_2}{2}, l_4uv_1 - \frac{\sqrt{2}fw_1}{2}, l_5uw_1 - \frac{\sqrt{2}fv_1}{2} \right) \\ \mathbb{H} &= \begin{pmatrix} 2l_2v_2^2 + \frac{\sqrt{2}fuw_1}{2v_1} & -\frac{l_6v_2^2w_1}{v_1} + \frac{1}{2}\sqrt{2}fu & 2l_2v_1v_2 & l_6v_2w_1 \\ -\frac{l_6v_2^2w_1}{v_1} + \frac{1}{2}\sqrt{2}fu & \frac{2l_3v_2^2w_1^2}{v_1^2} + \frac{\sqrt{2}fuw_1}{2w_1} & -l_6v_2w_1 & -\frac{2l_3v_2w_1^2}{v_1} \\ 2l_2v_1v_2 & -l_6v_2w_1 & 2l_2v_1^2 + \frac{\sqrt{2}fuw_1}{2v_1} & l_6v_1w_1 - \frac{1}{2}\sqrt{2}fu \\ l_6v_2w_1 & -\frac{2l_3v_2w_1^2}{v_1} & l_6v_1w_1 - \frac{1}{2}\sqrt{2}fu & 2l_3w_1^2 + \frac{\sqrt{2}fuw_1}{2w_1} \end{pmatrix} \end{aligned}$$

As we can see ($M_{b_{11}} \propto 1/u$), so we can uncouple the matrix and then take one Goldstone boson. In addition again we have a hierarchy structure where;

$$M_{b_{66}} \gg E \gg \mathbb{H}$$

So we can consider the same procedure as for the matrix \mathbb{M}_a . Applying equations H.1 and H.2, is possible to make a block diagonalization to obtain $(\lambda_{\xi_{x3}})$ and a new submatrix $\tilde{\mathbb{H}}$. Uncoupling the resultant matrix with the first component, we have $(\lambda_{\zeta_{x3}})$;

$$\begin{aligned} M_{\zeta_{x3}}^2 &\rightarrow \lambda_{\zeta_{x3}} \approx 0 \\ M_{\xi_{x3}}^2 &\rightarrow \lambda_{\xi_{x3}} \approx 2l_1u^2 + \frac{\sqrt{2}fw_1}{2uv_1}|v|^2 \\ \tilde{\mathbb{H}} &= \begin{pmatrix} -\frac{l_a v_2^2}{2l_1} + \frac{f_1 u w_1}{2v_1} & \frac{1}{2} f_1 u + \frac{l_b v_2^2 w_1}{2l_1 v_1} & -\frac{l_a v_1 v_2}{2l_1} & -\frac{l_b v_2 w_1}{2l_1} \\ \frac{1}{2} f_1 u + \frac{l_b v_2^2 w_1}{2l_1 v_1} & \frac{f_1 u w_1}{2w_1} + \frac{l_c v_2^2 w_1^2}{2l_1 v_1^2} & \frac{l_b v_2 w_1}{2l_1} & -\frac{l_c v_2 w_1^2}{2l_1 v_1} \\ -\frac{l_a v_1 v_2}{2l_1} & \frac{l_b v_2 w_1}{2l_1} & -\frac{l_a v_1^2}{2l_1} + \frac{f_1 u w_1}{2v_1} & -\frac{1}{2} f_1 u - \frac{l_b v_1 w_1}{2l_1} \\ -\frac{l_b v_2 w_1}{2l_1} & -\frac{l_c v_2 w_1^2}{2l_1 v_1} & -\frac{1}{2} f_1 u - \frac{l_b v_1 w_1}{2l_1} & \frac{f_1 u w_1}{2w_1} + \frac{l_c w_1^2}{2l_1} \end{pmatrix} \end{aligned}$$

using,

$$f_1 = \sqrt{2}f \quad , \quad l_a = l_4^2 - 4l_1l_2 \quad , \quad -l_c = l_5^2 - 4l_1l_3 \quad , \quad l_b = l_4l_5 - 2l_1l_6$$

This matrix is singular and does not have any hierarchy structure so we must to diagonalize a (4×4) matrix, with the corresponding eigenvalues.

$$\tilde{\mathbb{H}} : \begin{cases} \lambda_9 = 0 & \text{(H.6a)} \\ \lambda_{10} = \frac{f}{\sqrt{2}} \left(\frac{u(v_1^2 + w_1^2)}{v_1 w_1} \right) & \text{(H.6b)} \\ \lambda_{11} = \frac{f_1 l_1 w_1^3 - (l_a v_1^2 (v_1^2 + v_2^2)) w_1 + (f_1 l_1 u v_1) w_1^2 + (l_c (v_1^2 + v_2^2)) w_1^3 - \mathcal{R}}{4 l_1 v_1^2 w_1} & \text{(H.6c)} \\ \lambda_{12} = \frac{f_1 l_1 w_1^3 - (l_a v_1^2 (v_1^2 + v_2^2)) w_1 + (f_1 l_1 u v_1) w_1^2 + (l_c (v_1^2 + v_2^2)) w_1^3 + \mathcal{R}}{4 l_1 v_1^2 w_1} & \text{(H.6d)} \end{cases}$$

where;

$$\mathcal{R} = \sqrt{\alpha_0 + \alpha_1 w_1 + \alpha_2 w_1^2 + \alpha_3 w_1^3 + \alpha_4 w_1^4 + \alpha_5 w_1^5 + \alpha_6 w_1^6} \quad \text{(H.7)}$$

$$\alpha_0 = (f_1 l_1 u w_1^3)^2 \quad \text{(H.8)}$$

$$\alpha_1 = 2\sqrt{\alpha_0} l_a v_1^2 (v_1^2 + v_2^2) \quad \text{(H.9)}$$

$$\alpha_2 = \frac{2\alpha_0}{v_1^2} + l_a^2 v_1^4 (v_1^2 + v_2^2)^2 \quad \text{(H.10)}$$

$$\alpha_3 = -2\sqrt{\alpha_0} (l_a - 4l_b - l_c) (v_1^2 + v_2^2) \quad \text{(H.11)}$$

$$\alpha_4 = \alpha_0 + 2 (2 l_b^2 + l_a l_c) v_1^2 (v_1^2 + v_2^2)^2 \quad \text{(H.12)}$$

$$\alpha_5 = -2\sqrt{\alpha_0} l_c (v_1^2 + v_2^2) \quad \text{(H.13)}$$

$$\alpha_6 = l_c^2 (v_1^2 + v_2^2)^2 \quad \text{(H.14)}$$

Taking out the factor α_0 from the root and considering the energy scales to vanish terms like $(u^{-n} \quad n = 1, 2, \dots)$ as we make with matrix $(\tilde{\mathbb{H}})$, we have;

$$\mathcal{R} = \sqrt{\alpha_0 \left(1 + w_1^4 + \frac{2w_1^2}{v_1^2} \right)} \approx f_1 l_1 u w_1^3 \left(1 + \frac{w_1^2}{v_1^2} + \frac{w_1^4}{2} \right) \quad \text{(H.15)}$$

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